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TO THE NON-LOCAL THEORY OF CHARGE – SPIN INTERACTION IN WAVES AND PARTICLES

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The theory of the charge – spin interaction in waves in the frame of non-local quantum hydrodynamics is considered. The electron charge inner structure is investigated using the non-local physical description. From calculations follow that electrons can be considered like charged balls (shortly CB model) which charges are concentrated mainly in the shell of these balls. The possible direction deviation of the spin momentum and the magnetic momentum is taken into account.

***Keywords**: foundations of the theory of transport processes, the theory of solitons, generalized hydrodynamic equations, foundations of quantum mechanics.*

In Schrödinger–Pauli quantum theory the electron can be theoretically considered as a bound state of chargon, spinon and orbiton. In particle physics, spin is an intrinsic form of angular momentum carried by elementary particles including electron. The orbiton is carrying the orbital degree of freedom and the chargon is carrying the charge. One of the often used models in condensed matter physics is the spin–charge separation in electrons in some materials in which they "split" into three independent particles, the spinon, orbiton and the chargon (or its antiparticle, the holon).

Usually the theory of spin–charge separation originates with the work of Sin-Itiro Tomonaga who developed an approximate method for treating one-dimensional interacting quantum systems [1]. The aim of the article consists in consideration of the spin – charge separation and interaction from position of the non-local quantum hydrodynamics. The article is organized as follows. In the definite sense this paper can be considered as the prolongation of the article [2]. As result in Introduction (Section 1) the basic principles of generalized quantum hydrodynamics (GQH) created by me and expounded in particular in [3–8] are delivered in a brief form. As it was shown earlier the theory of transport processes (including quantum mechanics) can be applied in the frame of the unified theory based on the non-local physical description. In particular the generalized hydrodynamic equations represent an effective tool for solving problems in the very vast area of physical problems [9–12]. In Section 2 the system of non-local quantum hydrodynamic equations is applied for investigation of the charge – spin waves investigations, taking as a case in point the waves in graphene. Section 3 contains the basic non-local equations in spherical coordinate system for description of a negative charged physical system placed in a bounded region of a space. Internal energy ε_α of this one species object and a possible influence of the magnetic field are taken into account. In Section 4 is pointed out the important particular non-stationary one dimensional case corresponding to the negative charged system evolution in the potential electric field. The derivation of the angle relaxation equation is realized for the angle reflecting the possible deviation between a separated direction of the spin at the initial time moment and the direction of magnetic momentum after an external perturbation. Section 5 involves the mathematical modeling of the charge distribution in electron.

1. Introduction. About the basic principles of the generalized quantum hydrodynamics

Let us consider the transport processes in open dissipative systems and ideas of following transformation of generalized hydrodynamic description in quantum hydrodynamics which can be applied to the individual particle.

The kinetic description is inevitably related to the system diagnostics. Such an element of diagnostics in the case of theoretical description in physical kinetics is the concept of the physically infinitely small volume $(\mathbf{PhSV})$. The correlation between theoretical description and system diagnostics is well-known in physics. Suffice it to recall the part played by test charge in electrostatics or by test circuit in the physics of magnetic phenomena. The traditional definition of **PhSV** contains the statement to the effect that the **PhSV** contains a sufficient number of particles for introducing a statistical description; however, at the same time, the **PhSV** is much smaller than the volume *V* of the physical system under consideration; in a first approximation, this leads to the local approach in investigating of the transport processes. It is assumed in classical hydrodynamics that local thermodynamic equilibrium is first established within the **PhSV**, and only after that the transition occurs to global thermodynamic equilibrium if it is at all possible for the system under study.

Let us consider the hydrodynamic description in more detail from this point of view. Assume that we have two neighboring physically infinitely small volumes $\mathbf{PhSV_1}$ and $\mathbf{PhSV_2}$ in a non-equilibrium system. Even the point-like particles (starting after the last collision near the boundary between two mentioned volumes) can change the distribution functions in the neighboring volume. The adjusting of the particles dynamic characteristics for translational degrees of freedom takes several collisions in the simplest case. As result, we have in the definite sense "the Knudsen layer" between these volumes. This fact unavoidably leads to fluctuations in mass and hence in other hydrodynamic quantities. Existence of such "Knudsen layers" is not connected with the choice of space nets and fully defined by the reduced description for ensemble of particles of finite diameters in the conceptual frame of open physically small volumes, therefore – with the chosen method of measurement. This entire complex of effects defines non-local effects in space and time.

The physically infinitely small volume (**PhSV**) is an open thermodynamic system for any division of macroscopic system by a set of PhSVs. But the Boltzmann equation (BE) [3, 13, 14]

$$Df/Dt = J^B, \tag{1.1}$$

where J^B is the Boltzmann collision integral and D/Dt is a substantive derivative, fully ignores non-local effects and contains only the local collision integral J^B. The foregoing nonlocal effects are insignificant only in equilibrium systems, where the kinetic approach changes to methods of statistical mechanics.

This is what the difficulties of classical Boltzmann physical kinetics arise from. Also a weak point of the classical Boltzmann kinetic theory is the treatment of the dynamic properties of interacting particles. On the one hand, as follows from the so-called "physical" derivation of BE, Boltzmann particles are regarded as material points; on the other hand, the collision integral in the BE leads to the emergence of collision cross sections.

Notice that the application of the above principles also leads to the modification of the system of Maxwell equations. While the traditional formulation of this system does not involve the continuity equation, its derivation explicitly employs the equation

$$\frac{\partial \rho^a}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{j}^a = 0, \tag{1.2}$$

where ρ^a is the charge per unit volume, and $\mathbf{j}^a$ is the current density, both calculated without accounting for the fluctuations. As a result, the system of Maxwell equations written in the standard notation, namely:

$$\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{B} = 0, \ \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{D} = \rho^a, \\ \frac{\partial}{\partial \mathbf{r}} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \ \frac{\partial}{\partial \mathbf{r}} \times \mathbf{H} = \mathbf{j}^a + \frac{\partial \mathbf{D}}{\partial t}. \tag{1.3}$$

contains

$$\rho^a = \rho - \rho^{fl}, \ \mathbf{j}^a = \mathbf{j} - \mathbf{j}^{fl}. \tag{1.4}$$

The ρ^{fl}, $\mathbf{j}^{fl}$ fluctuations calculated using the generalized Boltzmann equation are given, for example, in Ref. [4, 6, 8]. The violation of Bell's inequalities [15] is found for local statistical theories, and the transition to non-local description is inevitable.

The rigorous approach to derivation of kinetic equation relative to one-particle DF f (KE_f) is based on employing the hierarchy of Bogoliubov equations. Generally speaking, the structure of KE_f is as follows:

$$\frac{Df}{Dt} = J^B + J^{nl}, \tag{1.5}$$

where J^{nl} is the non-local integral term. An approximation for the second collision integral is suggested by me in generalized Boltzmann physical kinetics,

$$J^{nl} = \frac{D}{Dt}\left(\tau \frac{Df}{Dt}\right). \tag{1.6}$$

Here, τ is non-local relaxation parameter, in the simplest case – the mean time between collisions of particles, which is related in a hydrodynamic approximation with dynamical viscosity μ and pressure p,

$$\tau \ p = \Pi \mu, \tag{1.7}$$

where the factor Π is defined by the model of collision of particles: for neutral hard-sphere gas, Π =0.8 [16, 17]. All of the known methods of the kinetic equation derivation relative to one-particle DF lead to approximation (1.6), including the method of many scales, the method of correlation functions, and the iteration method.

In the general case, the parameter τ is the non-locality parameter; in quantum hydrodynamics, its magnitude is correlated with the "time-energy" uncertainty relation [9, 10].

Now we can turn our attention to the quantum hydrodynamic description of individual particles. The abstract of the classical Madelung's paper [18] contains only one phrase: "It is shown that the Schrödinger equation for one-electron problems can be transformed into the form of hydrodynamic equations". The following conclusion of principal significance can be done from the previous consideration [9, 10]:

1. Madelung's quantum hydrodynamics is equivalent to the Schrödinger equation (SE) and leads to the description of the quantum particle evolution in the form of Euler equation and continuity equation. Quantum Euler equation contains additional potential of non-local origin which can be written for example in the Bohm form.

2. SE is consequence of the Liouville equation as result of the local approximation of non-local equations.

3. Generalized Boltzmann physical kinetics leads to the strict approximation of non-local effects in space and time and after going to the local approximation leads to parameter τ, which on the quantum level corresponds to the uncertainty principle "time-energy".

4. Generalized hydrodynamic equations (GHE) lead to SE as a deep particular case of the generalized Boltzmann physical kinetics and therefore of non-local hydrodynamics.

In principle GHE needn't in using of the "time-energy" uncertainty relation for estimation of the value of the non-locality parameter τ. Moreover the "time-energy" uncertainty relation does not lead to the exact relations and from position of non-local physics is only the simplest estimation of the non-local effects. Really, let us consider two neighboring physically infinitely small volumes $\mathbf{PhSV_1}$ and $\mathbf{PhSV_2}$ in a non-equilibrium system. Obviously the time τ should tends to diminishing with increasing of the velocities u of particles invading in the nearest neighboring physically infinitely small volume ($\mathbf{PhSV_1}$ or $\mathbf{PhSV_2}$):

$$\tau = H/u^n . \tag{1.8}$$

But the value τ cannot depend on the velocity direction and naturally to tie τ with the particle kinetic energy, then

$$\tau = H/\left(mu^2\right), \tag{1.9}$$

where H is a coefficient of proportionality, which reflects the state of physical system. In the simplest case H is equal to Plank constant $\hbar$ and relation (1.8) becomes compatible with the Heisenberg relation. Possible approximations of τ – parameter in details in the monographs [8, 20, 21] are considered. But some remarks of the principal significance should be done.

It is known that Ehrenfest adiabatic theorem is one of the most important and widely studied theorems in Schrödinger quantum mechanics. It states that if we have a slowly changing Hamiltonian that depends on time, and the system is prepared in one of the instantaneous eigenstates of the Hamiltonian then the state of the system at any time is given by an the instantaneous eigenfunction of the Hamiltonian up to multiplicative phase factors.

The adiabatic theory can be naturally incorporated in generalized quantum hydro-dynamics based on local approximations of non-local terms. In the simplest case if ΔQ is the elementary heat quantity delivered for a system executing the transfer from one state (the corresponding time moment is t_{in}) to the next one (the time moment t_e) then

$$\Delta Q = \frac{1}{\tau} 2\delta\left(\overline{T}\tau\right), \tag{1.10}$$

where $\tau = t_e - t_{in}$ and $\overline{T}$ is the average kinetic energy. For adiabatic case Ehrenfest supposes that

$$2\overline{T}\tau = \Omega_1, \Omega_2, ... \tag{1.11}$$

where $\Omega_1, \Omega_2, ...$ are adiabatic invariants. Obviously for Plank's oscillator (compare with (1.9)):

$$2\overline{T}\tau = nh. \tag{1.12}$$

Then the adiabatic theorem and consequences of this theory deliver the general quantization conditions for non-local quantum hydrodynamics.

2. Generalized quantum hydrodynamic equations

Strict consideration leads to the following system of the generalized hydrodynamic equations (GHE) [4, 8] written in the generalized Euler form:
continuity equation for species α:

$$\frac{\partial}{\partial t}\left\{\rho_\alpha - \tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha \mathbf{v}_0)\right]\right\} + \frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho_\alpha \mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha \mathbf{v}_0\mathbf{v}_0) + \ddot{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right]\right\} = R_\alpha, \tag{2.1}$$

and continuity equation for mixture:

$$\frac{\partial}{\partial t}\left\{\rho - \sum_\alpha \tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha \mathbf{v}_0)\right]\right\} + \frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho \mathbf{v}_0 - \sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha \mathbf{v}_0\mathbf{v}_0) + \ddot{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right]\right\} = 0. \tag{2.2}$$

Momentum equation for species:

$$\frac{\partial}{\partial t}\left\{\rho_\alpha \mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot \rho_\alpha \mathbf{v}_0\mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \right.\right.$$
$$\left.\left. - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right]\right\} - \mathbf{F}_\alpha^{(1)}\left[\rho_\alpha - \tau_\alpha\left(\frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}}(\rho_\alpha \mathbf{v}_0)\right)\right] -$$
$$- \frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha \mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot \rho_\alpha \mathbf{v}_0\mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \right.\right.$$
$$\left.\left. - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right]\right\} \times \mathbf{B} + \frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho_\alpha \mathbf{v}_0\mathbf{v}_0 + p_\alpha \vec{\mathrm{I}} - \tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0\mathbf{v}_0 + \right.\right. \tag{2.3}$$
$$+ p_\alpha \vec{\mathrm{I}}) + \frac{\partial}{\partial \mathbf{r}}\cdot \rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0 + 2\vec{\mathrm{I}}\left(\frac{\partial}{\partial \mathbf{r}}\cdot(p_\alpha \mathbf{v}_0)\right) + \frac{\partial}{\partial \mathbf{r}}\cdot(\vec{\mathrm{I}} p_\alpha \mathbf{v}_0) -$$
$$\left.\left. - \mathbf{F}_\alpha^{(1)}\rho_\alpha \mathbf{v}_0 - \rho_\alpha \mathbf{v}_0 \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0 \times \mathbf{B}]\mathbf{v}_0 - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0[\mathbf{v}_0 \times \mathbf{B}]\right]\right\} =$$
$$= \int m_\alpha \mathbf{v}_\alpha J_\alpha^{st,el} d\mathbf{v}_\alpha + \int m_\alpha \mathbf{v}_\alpha J_\alpha^{st,inel} d\mathbf{v}_\alpha .$$

Generalized moment equation for mixture:

$$\frac{\partial}{\partial t}\left\{\rho \mathbf{v}_0 - \sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot \rho_\alpha \mathbf{v}_0\mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \right.\right.$$
$$\left.\left. - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right]\right\} - \sum_\alpha \mathbf{F}_\alpha^{(1)}\left[\rho_\alpha - \tau_\alpha\left(\frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}}(\rho_\alpha \mathbf{v}_0)\right)\right] -$$
$$- \sum_\alpha \frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha \mathbf{v}_0 - \tau_\alpha^{(0)}\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot \rho_\alpha \mathbf{v}_0\mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \right.\right.$$
$$\left.\left. - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right]\right\} \times \mathbf{B} + \frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho \mathbf{v}_0\mathbf{v}_0 + p\vec{\mathrm{I}} - \sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0\mathbf{v}_0 + \right.\right. \tag{2.4}$$
$$+ p_\alpha \vec{\mathrm{I}}) + \frac{\partial}{\partial \mathbf{r}}\cdot \rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0 + 2\vec{\mathrm{I}}\left(\frac{\partial}{\partial \mathbf{r}}\cdot(p_\alpha \mathbf{v}_0)\right) + \frac{\partial}{\partial \mathbf{r}}\cdot(\vec{\mathrm{I}} p_\alpha \mathbf{v}_0) -$$
$$\left.\left. - \mathbf{F}_\alpha^{(1)}\rho_\alpha \mathbf{v}_0 - \rho_\alpha \mathbf{v}_0 \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0 \times \mathbf{B}]\mathbf{v}_0 - \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0[\mathbf{v}_0 \times \mathbf{B}]\right]\right\} = 0.$$

Energy equation for component:

$$\frac{\partial}{\partial t}\left\{\frac{\rho_\alpha v_0^2}{2} + \frac{3}{2}p_\alpha + \varepsilon_\alpha n_\alpha - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2} + \frac{3}{2}p_\alpha + \varepsilon_\alpha n_\alpha\right) + \right.\right.$$
$$\left.\left. + \frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2 \mathbf{v}_0 + \frac{5}{2}p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0\right) - \mathbf{F}_\alpha^{(1)}\cdot \rho_\alpha \mathbf{v}_0\right]\right\} +$$
$$+ \frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho_\alpha v_0^2 \mathbf{v}_0 + \frac{5}{2}p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2 \mathbf{v}_0 + \right.\right.\right.$$
$$\left. + \frac{5}{2}p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0\right) + \frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2 \mathbf{v}_0\mathbf{v}_0 + \frac{7}{2}p_\alpha \mathbf{v}_0\mathbf{v}_0 + \frac{1}{2}p_\alpha v_0^2 \vec{\mathrm{I}} + \right.$$
$$\left. + \frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\vec{\mathrm{I}} + \varepsilon_\alpha n_\alpha \mathbf{v}_0\mathbf{v}_0 + \varepsilon_\alpha \frac{p_\alpha}{m_\alpha}\vec{\mathrm{I}}\right) - \rho_\alpha \mathbf{F}_\alpha^{(1)}\cdot \mathbf{v}_0\mathbf{v}_0 - p_\alpha \mathbf{F}_\alpha^{(1)}\cdot \vec{\mathrm{I}} - \tag{2.5}$$
$$- \frac{1}{2}\rho_\alpha v_0^2 \mathbf{F}_\alpha^{(1)} - \frac{3}{2}\mathbf{F}_\alpha^{(1)} p_\alpha - \frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0 \times \mathbf{B}] - \frac{5}{2}p_\alpha \frac{q_\alpha}{m_\alpha}[\mathbf{v}_0 \times \mathbf{B}] -$$
$$\left.\left. - \varepsilon_\alpha n_\alpha \frac{q_\alpha}{m_\alpha}[\mathbf{v}_0 \times \mathbf{B}] - \varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)}\right]\right\} - \rho_\alpha \mathbf{F}_\alpha^{(1)}\cdot \mathbf{v}_0 +$$
$$+ \tau_\alpha \mathbf{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot \rho_\alpha \mathbf{v}_0\mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha \vec{\mathrm{I}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - q_\alpha n_\alpha[\mathbf{v}_0 \times \mathbf{B}]\right] =$$
$$= \int\left(\frac{m_\alpha v_\alpha^2}{2} + \varepsilon_\alpha\right) J_\alpha^{st,el} d\mathbf{v}_\alpha + \int\left(\frac{m_\alpha v_\alpha^2}{2} + \varepsilon_\alpha\right) J_\alpha^{st,inel} d\mathbf{v}_\alpha .$$

and after summation the generalized energy equation for mixture:

$$\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p+\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)+\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\}+$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0+\mathbf{v}_0\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\right.\right.\right.$$
$$\left.+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\ddot{\mathrm{I}}+\right.$$
$$\left.+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\ddot{\mathrm{I}}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\ddot{\mathrm{I}}\right)-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}}- \tag{2.6}$$
$$-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-$$
$$\left.\left.-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}-\mathbf{v}_0\cdot\sum_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}+$$
$$+\sum_\alpha\tau_\alpha\mathbf{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha\ddot{\mathrm{I}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right]=0.$$

Here $\mathbf{F}_\alpha^{(1)}$ are the forces of the non-magnetic origin, $\mathbf{B}$ – magnetic induction, $\ddot{\mathrm{I}}$ – unit tensor, q_α – charge of the α-component particle, p_α – static pressure for α-component, ε_α – internal energy for the particles of α- component, $\mathbf{v}_0$ – hydrodynamic velocity for mixture. For calculations in the self-consistent electro-magnetic field the system of non-local Maxwell equations should be added (see (1.3)).

It is well known that basic Schrödinger equation (SE) of quantum mechanics firstly was introduced as a quantum mechanical postulate. The obvious next step should be done and was realized by E. Madelung in 1927 – the derivation of special hydrodynamic form of SE after introduction wave function Ψ as:

$$\Psi(x,y,z,t)=\alpha(x,y,z,t)\ e^{i\beta(x,y,z,t)}. \tag{2.7}$$

Using (2.7) and separating the real and imagine parts of SE one obtains:

$$\frac{\partial\alpha^2}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{\alpha^2\hbar}{m}\frac{\partial\beta}{\partial \mathbf{r}}\right)=0, \tag{2.8}$$

and Eq. (2.8) immediately transforms in continuity equation if the identifications in the Madelung's notations for density ρ and velocity $\mathbf{v}$

$$\rho=\alpha^2=\Psi\Psi*, \tag{2.9}$$

$$\mathbf{v}=\frac{\partial}{\partial \mathbf{r}}(\beta\hbar/m). \tag{2.10}$$

introduce in Eq. (2.8). Identification for velocity (2.10) is obvious because for 1D flow with const values $p,\ E_k$

$$v=\frac{\partial}{\partial x}(\beta\hbar/m)=\frac{\hbar}{m}\frac{\partial}{\partial x}\left[-\frac{1}{\hbar}(E_k t-px)\right]= \\ =\frac{1}{m}\frac{\partial}{\partial x}(px)=v_\phi, \tag{2.11}$$

where v_ϕ is phase velocity. The existence of the condition (2.10) means that the corresponding flow has potential:

$$\Phi=\beta\hbar/m. \tag{2.12}$$

As result two effective hydrodynamic equations take place:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho\mathbf{v})=0, \tag{2.13}$$

$$\frac{\partial\mathbf{v}}{\partial t}+\frac{1}{2}\frac{\partial}{\partial \mathbf{r}}v^2=-\frac{1}{m}\frac{\partial}{\partial \mathbf{r}}\left(U-\frac{\hbar^2}{2m}\frac{\Delta\alpha}{\alpha}\right). \tag{2.14}$$

But:

$$\frac{\Delta\alpha}{\alpha}=\frac{\Delta\alpha^2}{2\alpha^2}-\frac{1}{\alpha^2}\left(\frac{\partial\alpha}{\partial \mathbf{r}}\right)^2, \tag{2.15}$$

and the relation (2.15) transforms (2.14) in particular case of the Euler motion equation:

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}})\mathbf{v}=-\frac{1}{m}\frac{\partial}{\partial \mathbf{r}}U^*, \tag{2.16}$$

where introduced the efficient potential:

$$U^*=U-\frac{\hbar^2}{4m\rho}\left[\Delta\rho-\frac{1}{2\rho}\left(\frac{\partial\rho}{\partial \mathbf{r}}\right)^2\right]. \tag{2.17}$$

Additive quantum part of potential can be written in the so called Bohm form:

$$\frac{\hbar^2}{2m\sqrt{\rho}}\Delta\sqrt{\rho}=\frac{\hbar^2}{4m\rho}\left[\Delta\rho-\frac{1}{2\rho}\left(\frac{\partial\rho}{\partial \mathbf{r}}\right)^2\right]. \tag{2.18}$$

Then

$$U^*=U+U_{qu}=U-\frac{\hbar^2}{2m\sqrt{\rho}}\Delta\sqrt{\rho}= \\ =U-\frac{\hbar^2}{4m\rho}\left[\Delta\rho-\frac{1}{2\rho}\left(\frac{\partial\rho}{\partial \mathbf{r}}\right)^2\right]. \tag{2.19}$$

Some remarks:

a) SE transforms in hydrodynamic form without additional assumptions. But numerical methods of hydrodynamics are very good developed. As result at the end of seventieth of the last century we realized the systematic calculations of quantum problems using quantum hydrodynamics (see for example [3, 19].

b) SE reduces to the system of continuity equation and the particular case of the Euler equation with the additional potential proportional to $\hbar^2$. The physical sense and the origin of the Bohm potential are established later in [9, 10].

c) SE (obtained in the frame of the theory of classical complex variables) cannot contain the energy equation in principle. As result in many cases the palliative approach is used when for solution of dissipative quantum problems the classical hydrodynamics is used with the insertion of the additional Bohm potential in the system of hydrodynamic equations.

d) The system of the generalized quantum hydrodynamic equations contains energy equation written for unknown dependent value which can be specified as quantum pressure p_α of non-local origin.

e) In chemically reaction systems the internal energies ε_α define the reactions heat Q. For example for bimolecular reaction $A_a + A_b \to A_c + A_d$ the reaction heat $\mathrm{Q} = \varepsilon_c + \varepsilon_d - \varepsilon_a - \varepsilon_b$.

f) For so called "elementary particles" the internal energy can contain the spin and magnetic parts. For example, electron has the internal energy ε:

$$\varepsilon_e = \varepsilon_{el,sp} + \varepsilon_{el,m}, \tag{2.20}$$

with the spin and magnetic parts, namely:

$$\varepsilon_{el,sp} = \hbar\omega/2,\ \varepsilon_{el,m} = -\mathbf{p_m}\cdot\mathbf{B}. \tag{2.21}$$

$\mathbf{p_m}$ – electron magnetic moment, $\mathbf{B}$ – magnetic induction. But $p_m = -\dfrac{e}{m_e}\dfrac{\hbar}{2c}$, then $\varepsilon_e = \dfrac{\hbar}{2}\omega_{eff}$.

The effective frequencies ω_{eff} can be altered in the process of the interaction with the surrounding environment. In this case the additional equations defining the change of the internal energies should be added to equations (2.1)–(2.6). Let us consider this situation in detail. I begin with case when the particle internal energy is constant.

After dividing the both sides of the continuity equation (2.1) by m_α and multiplying by ε_α this equation takes the form:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\varepsilon_\alpha n_\alpha - \tau_\alpha\left[\frac{\partial \varepsilon_\alpha n_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\cdot(\varepsilon_\alpha n_\alpha \mathbf{v}_0)\right]\right\} + \\
&+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\varepsilon_\alpha n_\alpha \mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}(\varepsilon_\alpha n_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot(\varepsilon_\alpha n_\alpha \mathbf{v}_0\mathbf{v}_0) + \right.\right. \\
&\left.\left.+\frac{1}{m_\alpha}\varepsilon_\alpha \overleftrightarrow{I}\cdot\frac{\partial p_\alpha}{\partial \mathbf{r}} - \varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\varepsilon_\alpha n_\alpha \mathbf{v}_0\times\mathbf{B}\right]\right\} = \\
&= \frac{1}{m_\alpha}\varepsilon_\alpha R_\alpha,
\end{aligned} \tag{2.22}$$

In general case if $\varepsilon_\alpha \neq const$ equation (2.22) is the internal energy equation in which the right hand side of equation $\dfrac{1}{m_\alpha}\varepsilon_\alpha R_\alpha$ transforms into function $\mathrm{E}_\alpha(\varepsilon_\alpha)$. After subtraction of the both sides of equation (2.22) from the corresponding parts of equation (2.5) one obtains:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\frac{\rho_\alpha v_0^2}{2} + \frac{3}{2}p_\alpha - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2} + \frac{3}{2}p_\alpha\right) + \frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0 + \frac{5}{2}p_\alpha\mathbf{v}_0\right) - \mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\} + \\
&+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0 + \frac{5}{2}p_\alpha\mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0 + \frac{5}{2}p_\alpha\mathbf{v}_0\right) + \right.\right. \\
&+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0 + \frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0 + \frac{1}{2}p_\alpha v_0^2\overleftrightarrow{I} + \frac{5}{2}p_\alpha\mathbf{v}_0\right) + \\
&+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0 + \frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0 + \frac{1}{2}p_\alpha v_0^2\overleftrightarrow{I} + \frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\overleftrightarrow{I}\right) - \rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0 - p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\overleftrightarrow{I} - \\
&\left.\left.-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)} - \frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha - \frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}] - \frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]\right]\right\} - \\
&-\left\{\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0 - \tau_\alpha\left[\mathbf{F}_\alpha^{(1)}\cdot\left(\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha\overleftrightarrow{I} - \rho_\alpha\mathbf{F}_\alpha^{(1)} - q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right)\right]\right\} = \\
&= \int\frac{m_\alpha v_\alpha^2}{2}J_\alpha^{st,el}d\mathbf{v}_\alpha + \int\frac{m_\alpha v_\alpha^2}{2}J_\alpha^{st,inel}d\mathbf{v}_\alpha.
\end{aligned} \tag{2.23}$$

taking into account that:

$$\int \varepsilon_\alpha J_\alpha^{st,el} d\mathbf{v}_\alpha + \int \varepsilon_\alpha J_\alpha^{st,inel} d\mathbf{v}_\alpha = \varepsilon_\alpha \frac{R_\alpha}{m_\alpha}. \qquad (2.24)$$

Conclusion: In the case when the change of the species internal energies is absent as result of interaction with external media the solution of the full system of equations (2.1)–(2.6) can be reduced to the system (2.1)–(2.5), (2.23).

It is interesting to confirm this conclusion by the direct numerical calculation. With this aim let us consider the charge density waves which are periodic modulation of the conduction electron density. The movement of the soliton waves in graphene was considered in the mentioned article [2]. I remind shortly the problem formulation.

The effective charge is created due to interference of the induced electron waves and correlating potentials as result of the polarized modulation of atomic positions. Therefore in this approach the conduction in graphene convoys the transfer of the positive ($+e, m_p$) and negative ($-e, m_e$) charges. Let us formulate the problem in detail. The non-stationary 1D motion of the combined soliton is considered under influence of the self-consistent electric forces of the potential and non-potential origin. It was shown [2] that mentioned soliton can exists without a chemical bond formation. Introduce the coordinate system ($\xi = x - Ct$) moving along the positive direction of the x axis with the velocity $C = u_0$, which is equal to the phase velocity of this quantum object.

Let us find the soliton type solutions for the system of the generalized quantum equations for two species mixture. The graphene crystal lattice is 2D flat structure which is considered in the moving coordinate system ($\xi = x - u_0 t$, y). In the following we intend to apply generalized non-local quantum hydrodynamic equations (2.1)–(2.6) to the investigation of the charge density waves (CDW) in the frame of two species model which lead to the following dimensional equations [9, 10]:

Poisson equation for the self-consistent electric field:

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial^2 \varphi}{\partial y^2} = -4\pi e\left\{\left[n_p - \tau_p \frac{\partial}{\partial \xi}\left(n_p(u-u_0)\right)\right] - \left[n_e - \tau_e \frac{\partial}{\partial \xi}\left(n_e(u-u_0)\right)\right]\right\}. \qquad (2.25)$$

Continuity equation for the positive particles:

$$\frac{\partial}{\partial \xi}\left[\rho_p(u_0-u)\right] + \frac{\partial}{\partial \xi}\left\{\tau_p \frac{\partial}{\partial \xi}\left[\rho_p(u-u_0)^2\right]\right\} + \frac{\partial}{\partial \xi}\left\{\tau_p\left[\frac{\partial}{\partial \xi}p_p - \rho_p F_{p\xi}\right]\right\} + \frac{\partial}{\partial y}\left\{\tau_p\left[\frac{\partial}{\partial y}p_p - \rho_p F_{py}\right]\right\} = 0. \qquad (2.26)$$

Continuity equation for electrons:

$$\frac{\partial}{\partial \xi}\left[\rho_e(u_0-u)\right] + \frac{\partial}{\partial \xi}\left\{\tau_e \frac{\partial}{\partial \xi}\left[\rho_e(u-u_0)^2\right]\right\} + \frac{\partial}{\partial \xi}\left\{\tau_e\left[\frac{\partial}{\partial \xi}p_e - \rho_e F_{e\xi}\right]\right\} + \frac{\partial}{\partial y}\left\{\tau_e\left[\frac{\partial}{\partial y}p_e - \rho_e F_{ey}\right]\right\} = 0. \qquad (2.27)$$

Momentum equation for the x direction:

$$\begin{aligned}
&\frac{\partial}{\partial \xi}\{\rho u(u-u_0) + p\} - \rho_p F_{p\xi} - \rho_e F_{e\xi} + \\
&+\frac{\partial}{\partial \xi}\left\{\tau_p\left[\frac{\partial}{\partial \xi}\left(2p_p(u_0-u) - \rho_p u(u_0-u)^2\right) - \rho_p F_{p\xi}(u_0-u)\right]\right\} + \\
&+\frac{\partial}{\partial \xi}\left\{\tau_e\left[\frac{\partial}{\partial \xi}\left(2p_e(u_0-u) - \rho_e u(u_0-u)^2\right) - \rho_e F_{e\xi}(u_0-u)\right]\right\} + \\
&+\tau_p F_{p\xi}\left(\frac{\partial}{\partial \xi}\left(\rho_p(u-u_0)\right)\right) + \tau_e F_{e\xi}\left(\frac{\partial}{\partial \xi}\left(\rho_e(u-u_0)\right)\right) - \\
&-\frac{\partial}{\partial \xi}\left\{\tau_p \frac{\partial}{\partial \xi}(p_p u)\right\} - \frac{\partial}{\partial \xi}\left\{\tau_e \frac{\partial}{\partial \xi}(p_e u)\right\} - \frac{\partial}{\partial y}\left\{\tau_p \frac{\partial}{\partial y}(p_p u)\right\} - \frac{\partial}{\partial y}\left\{\tau_e \frac{\partial}{\partial y}(p_e u)\right\} + \\
&+\frac{\partial}{\partial \xi}\left\{\tau_p\left[F_{p\xi}\rho_p u\right]\right\} + \frac{\partial}{\partial \xi}\left\{\tau_e\left[F_{e\xi}\rho_e u\right]\right\} + \frac{\partial}{\partial y}\left\{\tau_p\left[F_{py}\rho_p u\right]\right\} + \frac{\partial}{\partial y}\left\{\tau_e\left[F_{ey}\rho_e u\right]\right\} = 0.
\end{aligned} \qquad (2.28)$$

Energy equation for the positive particles:

$$\frac{\partial}{\partial \xi}\left[\rho_p u^2(u-u_0)+2\varepsilon_p n_p(u-u_0)+5p_p u-3p_p u_0\right]-2\rho_p F_{p\xi}u+$$
$$+\frac{\partial}{\partial \xi}\left\{\tau_p\left[\begin{array}{l}\frac{\partial}{\partial \xi}\left(-\rho_p u^2(u_0-u)^2-2\varepsilon_p n_p(u_0-u)^2+7p_p u(u_0-u)+3p_p u_0(u-u_0)-\right.\\ \left.-p_p u^2-2\varepsilon_p\frac{p_p}{m_p}-5\frac{p_p^2}{\rho_p}\right)-2F_{p\xi}\rho_p u(u_0-u)+\rho_p u^2 F_{p\xi}+2\varepsilon_p n_p F_{p\xi}+5p_p F_{p\xi}\end{array}\right]\right\}-$$
$$-\frac{\partial}{\partial y}\left\{\tau_p\left[\frac{\partial}{\partial y}\left(p_p u^2+2\varepsilon_p\frac{p_p}{m_p}+5\frac{p_p^2}{\rho_p}\right)-\rho_p F_{py}u^2-2\varepsilon_p n_p F_{py}-5p_p F_{py}\right]\right\}- \tag{2.29}$$
$$-2\tau_p F_{p\xi}\left[\frac{\partial}{\partial \xi}(\rho_p u(u_0-u))\right]-2\tau_p\rho_p\left[(F_{p\xi})^2+(F_{py})^2\right]+$$
$$+2\tau_p F_{p\xi}\left[\frac{\partial}{\partial \xi}p_p\right]+2\tau_p F_{py}\left[\frac{\partial}{\partial y}p_p\right]=-\frac{p_p-p_e}{\tau_{ep}}.$$

Energy equation for electrons:

$$\frac{\partial}{\partial \xi}\left[\rho_e u^2(u-u_0)+2\varepsilon_e n_e(u-u_0)+5p_e u-3p_e u_0\right]-2\rho_e F_{e\xi}u+$$
$$+\frac{\partial}{\partial \xi}\left\{\tau_e\left[\begin{array}{l}\frac{\partial}{\partial \xi}\left(-\rho_e u^2(u_0-u)^2-2\varepsilon_e n_e(u_0-u)^2+7p_e u(u_0-u)+3p_e u_0(u-u_0)-\right.\\ \left.-p_e u^2-2\varepsilon_e\frac{p_e}{m_e}-5\frac{p_e^2}{\rho_e}\right)-2F_{e\xi}\rho_e u(u_0-u)+\rho_e u^2 F_{e\xi}+2\varepsilon_e n_e F_{e\xi}+5p_e F_{e\xi}\end{array}\right]\right\}-$$
$$-\frac{\partial}{\partial y}\left\{\tau_e\left[\frac{\partial}{\partial y}\left(p_e u^2+2\varepsilon_e\frac{p_e}{m_e}+5\frac{p_e^2}{\rho_e}\right)-\rho_e F_{ey}u^2-2\varepsilon_e n_e F_{ey}-5p_e F_{ey}\right]\right\}- \tag{2.30}$$
$$-2\tau_e F_{e\xi}\left[\frac{\partial}{\partial \xi}(\rho_e u(u_0-u))\right]-2\tau_e\rho_e\left[(F_{e\xi})^2+(F_{ey})^2\right]+$$
$$+2\tau_e F_{e\xi}\left[\frac{\partial}{\partial \xi}p_e\right]+2\tau_e F_{ey}\left[\frac{\partial}{\partial y}p_e\right]=-\frac{p_e-p_p}{\tau_{ep}}.$$

Let write down these equations in the dimensionless form (see also [2]), where dimensionless symbols are marked by tildes; introduce the scales:

$$u=u_0\tilde{u}\,,\ \ \xi=x_0\tilde{\xi}\,,\ \ y=x_0\tilde{y}\,,\ \ \varphi=\varphi_0\tilde{\varphi}\,,$$
$$\rho_e=\rho_0\tilde{\rho}_e\,,\ \rho_p=\rho_0\tilde{\rho}_p\,, \tag{2.31}$$

where u_0, x_0, φ_0, ρ_0 – scales for velocity, distance, potential and density. Let us introduce also

$p_p=\rho_0 V_{0p}^2\tilde{p}_p$, $p_e=\rho_0 V_{0e}^2\tilde{p}_e$, where V_{0p} and V_{0e} – the scales for thermal velocities for the electron and positive species;

$$F_p=\tilde{\mathrm{F}}_\mathrm{p}\frac{e\varphi_0}{m_p x_0}\,,F_e=\tilde{\mathrm{F}}_\mathrm{e}\frac{e\varphi_0}{m_e x_0}\,;$$
$$\tau_p=\frac{m_e x_0 H}{m_p u_0\tilde{u}^2}\,,\ \tau_e=\frac{x_0 H}{u_0\tilde{u}^2}\,, \tag{2.32}$$

where $H=\frac{N_R\hbar}{m_e x_0 u_0}$ is dimensionless parameter.

Then $\frac{1}{\tau_{ep}}=\frac{u_0}{x_0}\frac{\tilde{u}^2}{H}\left(1+\frac{m_p}{m_e}\right)$.

Let us introduce also the following dimensionless parameters:

$$R=\frac{e\rho_0 x_0^2}{m_e\varphi_0},\ E=\frac{\mathrm{e}\varphi_0}{m_e u_0^2}\,. \tag{2.33}$$

and dimensionless parameters characterizing the internal particles energy:

$$S_e=\frac{2\varepsilon_e}{m_e u_0^2}\,,\ S_p=\frac{2\varepsilon_p}{m_p u_0^2}\,. \tag{2.34}$$

Acting forces are the sum of three terms: the self-consistent potential force (scalar potential φ), connected with the displacement of positive and negative charges, potential forces originated by the graphene crystal lattice (potential U) and the external electrical field creating the intensity **E**. As result the following relations are valid:

$$F_{p\xi}=\frac{e}{m_p}\left(-\frac{\partial\varphi}{\partial\xi}-\frac{\partial U}{\partial\xi}+E_{0\xi}\right),\ F_{e\xi}=\frac{e}{m_e}\left(\frac{\partial\varphi}{\partial\xi}+\frac{\partial U}{\partial\xi}-E_{0\xi}\right), \tag{2.35}$$

$$F_{py} = \frac{e}{m_p}\left(-\frac{\partial \varphi}{\partial y} - \frac{\partial U}{\partial y} + E_{0y}\right),\ F_{ey} = \frac{e}{m_e}\left(\frac{\partial \varphi}{\partial y} + \frac{\partial U}{\partial y} - E_{0y}\right), \tag{2.36}$$

or in the dimensionless form:

$$\tilde{\mathrm{F}}_{\mathrm{p}\xi} = -\frac{\partial \tilde\varphi}{\partial \tilde\xi} - \frac{\partial \tilde U}{\partial \tilde\xi} + \tilde E_\xi,\ \tilde{\mathrm{F}}_{\mathrm{e}\xi} = \frac{\partial \tilde\varphi}{\partial \tilde\xi} + \frac{\partial \tilde U}{\partial \tilde\xi} - \tilde E_\xi,\ \tilde{\mathrm{F}}_{\mathrm{py}} = -\frac{\partial \tilde\varphi}{\partial \tilde y} - \frac{\partial \tilde U}{\partial \tilde y} + \tilde E_y,\ \tilde{\mathrm{F}}_{\mathrm{ey}} = \frac{\partial \tilde\varphi}{\partial \tilde y} + \frac{\partial \tilde U}{\partial \tilde y} - \tilde E_y. \tag{2.37}$$

Graphene is a single layer of carbon atoms densely packed in a honeycomb lattice.

Taking into account the introduced values and approximations acting forces along y - direction for graphene (all details of the corresponding approximations are delivered in [2]) the following system of dimensionless non-local hydrodynamic equations for the 2D soliton description can be written in the first approximation:

Poisson equation for the self-consistent electric field:

$$\frac{\partial^2 \tilde\varphi}{\partial \tilde\xi^2} = -4\pi R\left\{\frac{m_e}{m_p}\left[\tilde\rho_p - \frac{m_e H}{m_p \tilde u^2}\frac{\partial}{\partial \tilde\xi}\left(\tilde\rho_p(\tilde u - 1)\right)\right] - \left[\tilde\rho_e - \frac{H}{\tilde u^2}\frac{\partial}{\partial \tilde\xi}\left(\tilde\rho_e(\tilde u - 1)\right)\right]\right\}. \tag{2.38}$$

Continuity equation for the positive particles:

$$\begin{aligned}&\frac{\partial}{\partial \tilde\xi}\left[\tilde\rho_p(1-\tilde u)\right] + \frac{m_e}{m_p}\frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\frac{\partial}{\partial \tilde\xi}\left[\tilde\rho_p(\tilde u - 1)^2\right]\right\} + \frac{m_e}{m_p}\frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\left[\frac{V_{0p}^2}{u_0^2}\frac{\partial}{\partial \tilde\xi}\tilde p_p - \right.\right.\\ &\left.\left.-\frac{m_e}{m_p}\tilde\rho_p E\left(-\frac{\partial \tilde\varphi}{\partial \tilde\xi} + \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\right]\right\} = 0.\end{aligned} \tag{2.39}$$

Continuity equation for electrons:

$$\begin{aligned}&\frac{\partial}{\partial \tilde\xi}\left[\tilde\rho_e(1-\tilde u)\right] + \frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\frac{\partial}{\partial \tilde\xi}\left[\tilde\rho_e(\tilde u - 1)^2\right]\right\} + \frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\left[\frac{V_{0e}^2}{u_0^2}\frac{\partial}{\partial \tilde\xi}\tilde p_e - \right.\right.\\ &\left.\left.-\tilde\rho_e E\left(\frac{\partial \tilde\varphi}{\partial \tilde\xi} - \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\right]\right\} = 0.\end{aligned} \tag{2.40}$$

Momentum equation for the x direction:

$$\begin{aligned}&\frac{\partial}{\partial \tilde\xi}\left\{(\tilde\rho_p + \tilde\rho_e)\tilde u(\tilde u - 1) + \frac{V_{0p}^2}{u_0^2}\tilde p_p + \frac{V_{0e}^2}{u_0^2}\tilde p_e\right\} - \frac{m_e}{m_p}\tilde\rho_p E\left(-\frac{\partial \tilde\varphi}{\partial \tilde\xi} + \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right) - \\ &-\tilde\rho_e E\left(\frac{\partial \tilde\varphi}{\partial \tilde\xi} - \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right) + \frac{m_e}{m_p}\frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\left[\frac{\partial}{\partial \tilde\xi}\left(2\frac{V_{0p}^2}{u_0^2}\tilde p_p(1-\tilde u) - \tilde\rho_p\tilde u(1-\tilde u)^2\right) - \right.\right.\\ &\left.\left.-\frac{m_e}{m_p}\tilde\rho_p(1-\tilde u)E\left(-\frac{\partial \tilde\varphi}{\partial \tilde\xi} + \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\right]\right\} + \\ &+\frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\left[\frac{\partial}{\partial \tilde\xi}\left(2\frac{V_{0e}^2}{u_0^2}\tilde p_e(1-\tilde u) - \tilde\rho_e\tilde u(1-\tilde u)^2\right) - \tilde\rho_e(1-\tilde u)E\left(\frac{\partial \tilde\varphi}{\partial \tilde\xi} - \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\right]\right\} + \\ &+\frac{H}{\tilde u^2}E\left(\frac{m_e}{m_p}\right)^2\left(-\frac{\partial \tilde\varphi}{\partial \tilde\xi} + \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\left(\frac{\partial}{\partial \tilde\xi}\left(\tilde\rho_p(\tilde u - 1)\right)\right) + \\ &+\frac{H}{\tilde u^2}E\left(\frac{\partial \tilde\varphi}{\partial \tilde\xi} - \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\left(\frac{\partial}{\partial \tilde\xi}\left(\tilde\rho_e(\tilde u - 1)\right)\right) - \\ &-\frac{m_e}{m_p}\frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\frac{V_{0p}^2}{u_0^2}\frac{\partial}{\partial \tilde\xi}\left(\tilde p_p\tilde u\right)\right\} - \frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\frac{V_{0e}^2}{u_0^2}\frac{\partial}{\partial \tilde\xi}\left(\tilde p_e\tilde u\right)\right\} + \\ &+\left(\frac{m_e}{m_p}\right)^2 E\frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\left[\left(-\frac{\partial \tilde\varphi}{\partial \tilde\xi} + \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\tilde\rho_p\tilde u\right]\right\} + \\ &+E\frac{\partial}{\partial \tilde\xi}\left\{\frac{H}{\tilde u^2}\left[\left(\frac{\partial \tilde\varphi}{\partial \tilde\xi} - \tilde U'_{11}\sin\left(\frac{4\pi}{3\tilde a}\tilde\xi - \frac{\pi}{3}\right)\right)\tilde\rho_e\tilde u\right]\right\} = 0.\end{aligned} \tag{2.41}$$

Energy equation for the positive particles:

$$
\begin{aligned}
&\frac{\partial}{\partial\tilde{\xi}}\left[\tilde{\rho}_p\tilde{u}^2(\tilde{u}-1)+S_p\tilde{\rho}_p(\tilde{u}-1)+5\frac{V_{0p}^2}{u_0^2}\tilde{p}_p\tilde{u}-3\frac{V_{0p}^2}{u_0^2}\tilde{p}_p\right]-\\
&-2\frac{m_e}{m_p}\tilde{\rho}_p E\left(-\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}+\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)\tilde{u}+\\
&+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{H}{\tilde{u}^2}\frac{m_e}{m_p}\left[\frac{\partial}{\partial\tilde{\xi}}\left(-\tilde{\rho}_p\tilde{u}^2(1-\tilde{u})^2-S_p\tilde{\rho}_p(\tilde{u}-1)^2+7\frac{V_{0p}^2}{u_0^2}\tilde{p}_p\tilde{u}(1-\tilde{u})+\right.\right.\right.\\
&\left.+3\frac{V_{0p}^2}{u_0^2}\tilde{p}_p(\tilde{u}-1)-\frac{V_{0p}^2}{u_0^2}\tilde{p}_p\tilde{u}^2-\frac{V_{0p}^2}{u_0^2}S_p\tilde{p}_p-5\frac{V_{0p}^4}{u_0^4}\frac{\tilde{p}_p^2}{\tilde{\rho}_p}\right)+\\
&+E\left(-2\frac{m_e}{m_p}\tilde{\rho}_p\tilde{u}(1-\tilde{u})+\frac{m_e}{m_p}\tilde{\rho}_p\tilde{u}^2+\frac{m_e}{m_p}S_p\tilde{\rho}_p+5\frac{m_e}{m_p}\frac{V_{0p}^2}{u_0^2}\tilde{p}_p\right)\left(-\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}+\right.\\
&\left.\left.\left.+\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)\right]\right\}+2\frac{H}{\tilde{u}^2}E\left(\frac{m_e}{m_p}\right)^2\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_p\tilde{u}(1-\tilde{u})\right)+\right.\\
&\left.+\frac{V_{0p}^2}{u_0^2}\frac{\partial}{\partial\tilde{\xi}}\tilde{p}_p\right]\left(-\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}+\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)-\\
&-2\frac{H}{\tilde{u}^2}E^2\left(\frac{m_e}{m_p}\right)^3\tilde{\rho}_p\left[\left(-\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}+\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)^2+\frac{1}{2}\left(\tilde{U}'_{10}\sin\left(\frac{2\pi}{3\tilde{a}}\tilde{\xi}+\frac{\pi}{3}\right)\right)^2+\right.\\
&\left.+\frac{3}{2}\left(\tilde{U}'_{10}\cos\left(\frac{2\pi}{3\tilde{a}}\tilde{\xi}+\frac{\pi}{3}\right)\right)^2+6\left(\tilde{U}'_{11}\right)^2+\frac{16}{\pi}\left(\tilde{U}'_{10}\tilde{U}'_{11}\right)\cos\left(\frac{2\pi}{3\tilde{a}}\tilde{\xi}+\frac{\pi}{3}\right)\right]=\\
&=-\frac{\tilde{u}^2}{Hu_0^2}\left(V_{0p}^2\tilde{p}_p-\tilde{p}_eV_{0e}^2\right)\left(1+\frac{m_p}{m_e}\right).
\end{aligned}
\tag{2.42}
$$

Energy equation for electrons:

$$
\begin{aligned}
&\frac{\partial}{\partial\tilde{\xi}}\left[\tilde{\rho}_e\tilde{u}^2(\tilde{u}-1)+S_e\tilde{\rho}_e(\tilde{u}-1)+5\frac{V_{0e}^2}{u_0^2}\tilde{p}_e\tilde{u}-3\frac{V_{0e}^2}{u_0^2}\tilde{p}_e\right]-2\tilde{\rho}_e\tilde{u}E\left(\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}-\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)+\\
&+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{H}{\tilde{u}^2}\left[\frac{\partial}{\partial\tilde{\xi}}\left(-\tilde{\rho}_e\tilde{u}^2(1-\tilde{u})^2-S_e\tilde{\rho}_e(\tilde{u}-1)^2+7\frac{V_{0e}^2}{u_0^2}\tilde{p}_e\tilde{u}(1-\tilde{u})+3\frac{V_{0e}^2}{u_0^2}\tilde{p}_e(\tilde{u}-1)-\frac{V_{0e}^2}{u_0^2}\tilde{p}_e\tilde{u}^2-\right.\right.\right.\\
&\left.\left.\left.-\frac{V_{0e}^2}{u_0^2}S_e\tilde{p}_e-5\frac{V_{0e}^4}{u_0^4}\frac{\tilde{p}_e^2}{\tilde{\rho}_e}\right)+E\left(-2\tilde{\rho}_e\tilde{u}(1-\tilde{u})+\tilde{\rho}_e\tilde{u}^2+S_e\tilde{\rho}_e+5\frac{V_{0e}^2}{u_0^2}\tilde{p}_e\right)\left(\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}-\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)\right]\right\}+\\
&+E\left(-2\frac{H}{\tilde{u}^2}\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}(1-\tilde{u})\right)+2\frac{H}{\tilde{u}^2}\frac{V_{0e}^2}{u_0^2}\frac{\partial}{\partial\tilde{\xi}}\tilde{p}_e\right)\left(\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}-\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)-\\
&-2E^2\frac{H}{\tilde{u}^2}\tilde{\rho}_e\left[\left(-\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}+\tilde{U}'_{11}\sin\left(\frac{4\pi}{3\tilde{a}}\tilde{\xi}-\frac{\pi}{3}\right)\right)^2+\frac{1}{2}\left(\tilde{U}'_{10}\sin\left(\frac{2\pi}{3\tilde{a}}\tilde{\xi}+\frac{\pi}{3}\right)\right)^2+6\left(\tilde{U}'_{11}\right)^2+\right.\\
&\left.+\frac{3}{2}\left(\tilde{U}'_{10}\cos\left(\frac{2\pi}{3\tilde{a}}\tilde{\xi}+\frac{\pi}{3}\right)\right)^2+\frac{16}{\pi}\left(\tilde{U}'_{10}\tilde{U}'_{11}\right)\cos\left(\frac{2\pi}{3\tilde{a}}\tilde{\xi}+\frac{\pi}{3}\right)\right]=-\frac{\tilde{u}^2}{Hu_0^2}\left(V_{0e}^2\tilde{p}_e-V_{0p}^2\tilde{p}_p\right)\left(1+\frac{m_p}{m_e}\right).
\end{aligned}
\tag{2.43}
$$

The calculations are realized on the basement of equations (2.38)–(2.43) by the initial conditions and parameters containing in the Table 1. The vast results of the mathematical modeling realized with the help of Maple (the versions Maple 9 or more can be used) can be found in [2]. Here I discuss only the calculations of the mentioned Variant 1.

The following Maple notations on figures are used: r – density $\tilde{\rho}_p$, s – density $\tilde{\rho}_e$, u – velocity $\tilde{u}$, p – pressure $\tilde{p}_p$, q – pressure $\tilde{p}_e$ and v – self consistent potential $\tilde{\varphi}$. Explanations placed under all following figures, Maple program contains Maple's notations – for example, the expression $D(u)(0)=0$ means in the usual notations $\frac{\partial\tilde{\mathrm{u}}}{\partial\tilde{\xi}}(0)=0$, independent variable t responds to $\tilde{\xi}$.

The solution exists only in the restricted domain of the 1D space and the obtained object in the moving coordinate system ($\tilde{\xi}=\tilde{x}-\tilde{t}$) has the constant velocity $\tilde{u}=1$ for all parts of the object. In this case the domain of the solution existence defines the character soliton size. The following numerical results (Table 2) demonstrate the realization of mentioned principles. Figures 1, 2 reflect the result of calculations for Variant 1 (Table 1) in the first approximation.

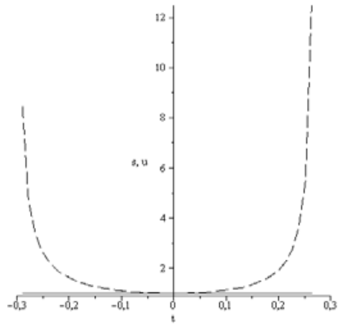

Fig. 1. s – the electron density $\tilde{\rho}_e$, u – velocity $\tilde{u}$ (solid line).

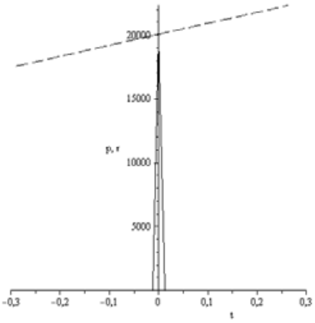

Fig. 2. r – the positive particles density, (solid line); p – the positive particles pressure.

Table 1. Initial conditions and parameters of calculations for Variant 1

$\tilde{a}$	L	T	$\tilde{\rho}_e(0)$	$\tilde{\rho}_p(0)$	$N=\frac{V_{0e}^2}{u_0^2}$	$P=\frac{V_{0p}^2}{u_0^2}$	$\tilde{p}_e(0)$	$\tilde{p}_p(0)$	$\tilde{\varphi}(0)$
1	1	$2\cdot10^4$	1	$2\cdot10^4$	1	10^{-4}	1	$2\cdot10^4$	1
$E=\frac{e\varphi_0}{m_e u_0^2}$	$R=\frac{e\rho_0 x_0^2}{m_e\varphi_0}$	H	$\frac{\partial\tilde{\rho}_e}{\partial\tilde{\xi}}(0)$	$\frac{\partial\tilde{\rho}_p}{\partial\tilde{\xi}}(0)$	$\frac{\partial\tilde{p}_e}{\partial\tilde{\xi}}(0)$	$\frac{\partial\tilde{p}_p}{\partial\tilde{\xi}}(0)$	$K=\frac{\partial\tilde{\varphi}}{\partial\tilde{\xi}}(0)$	$\tilde{U}'_{10}$	$\tilde{U}'_{11}$
0.1	0.003	15	0	0	0	0	0	10	10

Table 2. Numerical results of calculations for Variant 1($S_e=S_p=0$)

	t = $\tilde{\xi}$ = 0.2	t = $\tilde{\xi}$ = 0.25
p = $\tilde{p}_p$	21731.595	22164.607
p′ = $\partial\tilde{p}_p/\partial\tilde{\xi}$	8660.254	8660.254
q = $\tilde{p}_e$	0.956424	0.925592
q′ = $\partial\tilde{p}_e/\partial\tilde{\xi}$	-0.476250	-0.859321
r = $\tilde{\rho}_p$	$0.622976\cdot10^{-3}$	$0.407662\cdot10^{-3}$
r′ = $\partial\tilde{\rho}_p/\partial\tilde{\xi}$	-0.593701	$-0.308647\cdot10^{-2}$
s = $\tilde{\rho}_e$	1.866551	4.681384
s′ = $\partial\tilde{\rho}_e/\partial\tilde{\xi}$	18.453042	170.620851
u = $\tilde{u}$	1.000000	1.000000
v = $\tilde{\varphi}$	1.000819	1.0013853
v′ = $\partial\tilde{\varphi}/\partial\tilde{\xi}$	$0.909275\cdot10^{-2}$	$0.143210\cdot10^{-1}$

Now I can formulate some principal conclusions:

1. All calculations realized as Variant 1 and containing in Table 2 correspond to spin variables $S_e=S_p=0$. The domain of the soliton existence is equal to $\tilde{\xi}$ varying in interval (-0.305, 0.274).
2. All calculations realized as Variant 1 corresponding to constant spin variables S_e, S_p varying from $S_e=S_p=0$ to $S_e=S_p=10^9$ lead to the absolutely the same results shown in Table 2. The domain of the soliton existence is also equal to (-0.305, 0.274).
3. This fact confirms the previous theoretical result - in the case when the change of the species internal energies is absent as result of interaction with external media, the solution of the full system of equations (2.1)–(2.6) can be reduced to the system (2.1)–(2.5), (2.23).
4. These calculations realized by several numerical methods are the direct evidence in favor of high accuracy of numerical methods in the interactive Maple system for solution of the ordinary differential equations.

3. The charge internal structure of electron

Let us consider a negative charged physical system placed in a bounded region of a space. Internal energy ε_α of this one species object and a possible influence of the magnetic field are taken into account. The character linear scale of this region will be defined as result of the self-consistent solution of the generalized non-local quantum hydrodynamic equations (2.1)–(2.6). In the following I intend to suppose also that the mentioned physical object for simplicity has the spherical form and the system (2.1)–(2.6) is reasonable to write in the spherical coordinate system [20, 21]. Remark also that the terms ρg_r, ρg_θ, ρg_φ correspond to the components of the mass forces acting on the unit of volume. For example, for the potential forces of the electrical origin $\rho g_r = m_e n g_r = -m_e n\frac{eE}{m_e} = -neE = q\frac{\partial\psi}{\partial r}$.

It means also that in the following q is the absolute value of the negative charge per the unit of volume.

We have:

non-local continuity equation:

$$\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}\sin\theta\right)}{\partial\theta}\right]\right\}+$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_{0r}-\tau\left[\frac{\partial}{\partial t}(\rho v_{0r})+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0r}\right)}{\partial\varphi}+\right.\right.\right.$$
$$\left.\left.\left.+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0r}\sin\theta\right)}{\partial\theta}-\rho g_r-\frac{q}{m}\rho\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)\right]\right\}\right\}+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left\{\rho v_{0\varphi}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\varphi}\right)+\right.\right.$$
$$\left.\left.+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\varphi}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}^2\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0\varphi}\sin\theta\right)}{\partial\theta}-\rho g_\varphi-\frac{q}{m}\rho\left(v_{0\theta}B_r-v_{0r}B_\theta\right)\right]\right\}+ \qquad (3.1)$$
$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\left\{\rho v_{0\theta}-\tau\left[\frac{\partial}{\partial t}(\rho v_{0\theta})+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\theta}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0\theta}\right)}{\partial\varphi}+\right.\right.\right.$$
$$\left.\left.\left.+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}^2\sin\theta\right)}{\partial\theta}-\rho g_\theta-\frac{q}{m}\rho\left(v_{0r}B_\varphi-v_{0\varphi}B_r\right)\right]\right\}\right\}-$$
$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial p}{\partial r}\right)-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\tau\sin\theta\frac{\partial p}{\partial\theta}\right)-\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(\tau\frac{\partial p}{\partial\varphi}\right)=0.$$

Non-local momentum equation ($\mathbf{e}_r$ projection):

$$\frac{\partial}{\partial t}\left\{\rho v_{0r}-\tau\left[\frac{\partial}{\partial t}(\rho v_{0r})+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0r}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0r}\sin\theta\right)}{\partial\theta}+\right.\right.$$
$$\left.\left.+\frac{\partial p}{\partial r}-\rho g_r-\frac{q}{m}\rho\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)\right]\right\}-$$
$$-g_r\left[\rho-\tau\left(\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}\sin\theta\right)}{\partial\theta}\right)\right]-$$
$$-\frac{q}{m}\left(\rho v_{0\varphi}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\varphi}\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\varphi}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}^2\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0\varphi}\sin\theta\right)}{\partial\theta}+\right.\right.$$
$$\left.\left.+\frac{1}{r\sin\theta}\frac{\partial p}{\partial\varphi}-\rho g_\varphi-\frac{q}{m}\rho\left(v_{0\theta}B_r-v_{0r}B_\theta\right)\right]\right)B_\theta+$$
$$+\frac{q}{m}\left(\rho v_{0\theta}-\tau\left[\frac{\partial}{\partial t}(\rho v_{0\theta})+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\theta}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0\theta}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}^2\sin\theta\right)}{\partial\theta}+\right.\right.$$
$$\left.\left.+\frac{1}{r}\frac{\partial p}{\partial\theta}-\rho g_\theta-\frac{q}{m}\rho\left(v_{0r}B_\varphi-v_{0\varphi}B_r\right)\right]\right)B_\varphi+$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_{0r}^2-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0r}^2\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^3\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0r}^2\right)}{\partial\varphi}+\right.\right.\right.$$
$$\left.\left.\left.+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0r}^2\sin\theta\right)}{\partial\theta}-2g_r\rho v_{0r}-2\frac{q}{m}\rho\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)\right]\right\}\right\}+ \qquad (3.2)$$
$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left\{\rho v_{0\varphi}v_{0r}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\varphi}v_{0r}\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0\varphi}v_{0r}^2\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}^2v_{0r}\right)}{\partial\varphi}+\right.\right.$$
$$+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0\varphi}v_{0r}\sin\theta\right)}{\partial\theta}-g_\varphi\rho v_{0r}-\frac{q}{m}\rho\left(v_{0\theta}B_r-v_{0r}B_\theta\right)v_{0r}-$$
$$\left.\left.-\frac{q}{m}\rho v_{0\varphi}\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)-v_{0\varphi}\rho g_r\right]\right\}+$$
$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\left\{\rho v_{0\theta}v_{0r}-\tau\left[\frac{\partial}{\partial t}(\rho v_{0\theta}v_{0r})+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0\theta}v_{0r}^2\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0\theta}v_{0r}\right)}{\partial\varphi}+\right.\right.\right.$$
$$+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}^2v_{0r}\sin\theta\right)}{\partial\theta}-g_\theta\rho v_{0r}-\frac{q}{m}\rho\left(v_{0r}B_\varphi-v_{0\varphi}B_r\right)v_{0r}-$$
$$\left.\left.\left.-v_{0\theta}\rho g_r-\frac{q}{m}\rho\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)v_{0\theta}\right]\right\}\right\}+$$
$$+\frac{\partial p}{\partial r}-\frac{\partial}{\partial r}\left(\tau\frac{\partial p}{\partial t}\right)-2\frac{\partial}{\partial r}\left(\tau\left(\frac{1}{r^2}\frac{\partial\left(r^2pv_{0r}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(pv_{0\varphi}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(pv_{0\theta}\sin\theta\right)}{\partial\theta}\right)\right)-$$
$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial\left(pv_{0r}\right)}{\partial r}\right)-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\tau\sin\theta\frac{\partial\left(pv_{0r}\right)}{\partial\theta}\right)-\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(\tau\frac{\partial\left(pv_{0r}\right)}{\partial\varphi}\right)=0.$$

Non-local momentum equation ($\mathbf{e}_\varphi$ projection):

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left\{\rho v_{0\varphi}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\varphi}\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\varphi}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}^2\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0\varphi}\sin\theta\right)}{\partial\theta}+\right.\right.\\
&\left.\left.+\frac{1}{r\sin\theta}\frac{\partial p}{\partial\varphi}-\rho g_\varphi-\frac{q}{m}\rho\left(v_{0\theta}B_r-v_{0r}B_\theta\right)\right]\right\}-\\
&-g_\varphi\left[\rho-\tau\left(\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}\sin\theta\right)}{\partial\theta}\right)\right]-\\
&-\frac{q}{m}\left(\rho v_{0\theta}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\theta}\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\theta}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0\theta}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}^2\sin\theta\right)}{\partial\theta}+\right.\right.\\
&\left.\left.+\frac{1}{r}\frac{\partial p}{\partial\theta}-\rho g_\theta-\frac{q}{m}\rho\left(v_{0r}B_\varphi-v_{0\varphi}B_r\right)\right]\right)B_r+\\
&+\frac{q}{m}\left(\rho v_{0r}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0r}\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}v_{0r}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0r}\sin\theta\right)}{\partial\theta}+\right.\right.\\
&\left.\left.+\frac{\partial p}{\partial r}-\rho g_r-\frac{q}{m}\rho\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)\right]\right)B_\theta+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_{0r}v_{0\varphi}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0r}v_{0\varphi}\right)+\right.\right.\right.\\
&+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2v_{0\varphi}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}^2v_{0r}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0r}v_{0\varphi}\sin\theta\right)}{\partial\varphi}-\\
&\left.\left.\left.-g_r\rho v_{0\varphi}-v_{0r}\rho g_\varphi-\frac{q}{m}\rho\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)v_{0\varphi}-\frac{q}{m}\rho\left(v_{0\theta}B_r-v_{0r}B_\theta\right)v_{0r}\right]\right\}\right\}+\\
&+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left\{\rho v_{0\varphi}^2-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\varphi}^2\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\varphi}^2\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}^3\right)}{\partial\varphi}+\right.\right.\\
&\left.\left.+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}v_{0\varphi}^2\sin\theta\right)}{\partial\theta}-2g_\varphi\rho v_{0\varphi}-2\frac{q}{m}\rho\left(v_{0\theta}B_r-v_{0r}B_\theta\right)v_{0\varphi}\right]\right\}+\\
&+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\left\{\rho v_{0\theta}v_{0\varphi}-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\theta}v_{0\varphi}\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}v_{0\theta}v_{0\varphi}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\varphi}^2v_{0\theta}\right)}{\partial\varphi}+\right.\right.\right.\\
&+\frac{1}{r\sin\theta}\frac{\partial\left(\rho v_{0\theta}^2v_{0\varphi}\sin\theta\right)}{\partial\theta}-g_\theta\rho v_{0\varphi}-\frac{q}{m}\rho\left(v_{0r}B_\varphi-v_{0\varphi}B_r\right)v_{0\varphi}-\\
&\left.\left.\left.-v_{0\theta}\rho g_\varphi-\frac{q}{m}\rho\left(v_{0\theta}B_r-v_{0r}B_\theta\right)v_{0\theta}\right]\right\}\right\}+\frac{1}{r\sin\theta}\frac{\partial p}{\partial\varphi}-\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\tau\frac{\partial p}{\partial t}\right)-\\
&-\frac{2}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\tau\left(\frac{1}{r^2}\frac{\partial\left(r^2pv_{0r}\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(pv_{0\varphi}\right)}{\partial\varphi}+\frac{1}{r\sin\theta}\frac{\partial\left(pv_{0\theta}\sin\theta\right)}{\partial\theta}\right)\right)-\\
&-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial\left(pv_{0\varphi}\right)}{\partial r}\right)-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\tau\sin\theta\frac{\partial\left(pv_{0\varphi}\right)}{\partial\theta}\right)-\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(\tau\frac{\partial\left(pv_{0\varphi}\right)}{\partial\varphi}\right)=0.
\end{aligned}
\tag{3.3}
$$

Non-local momentum equation ($\mathbf{e}_\theta$ projection):

$$
\begin{aligned}
&\frac{\partial}{\partial t}\Bigg\{\rho v_{0\theta} - \tau\Bigg[\frac{\partial}{\partial t}(\rho v_{0\theta}) + \frac{1}{r^2}\frac{\partial(r^2\rho v_{0r}v_{0\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\varphi}v_{0\theta})}{\partial\varphi} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\theta}^2\sin\theta)}{\partial\theta} + \\
&+ \frac{1}{r}\frac{\partial p}{\partial\theta} - \rho g_\theta - \frac{q}{m}\rho(v_{0r}B_\varphi - v_{0\varphi}B_r)\Bigg]\Bigg\} - \\
&- g_\theta\Bigg[\rho - \tau\Bigg(\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial(r^2\rho v_{0r})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\varphi})}{\partial\varphi} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\theta}\sin\theta)}{\partial\theta}\Bigg)\Bigg] - \\
&- \frac{q}{m}\Bigg(\rho v_{0r} - \tau\Bigg[\frac{\partial}{\partial t}(\rho v_{0r}) + \frac{1}{r^2}\frac{\partial(r^2\rho v_{0r}^2)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\varphi}v_{0r})}{\partial\varphi} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\theta}v_{0r}\sin\theta)}{\partial\theta} + \\
&+ \frac{\partial p}{\partial r} - \rho g_r - \frac{q}{m}\rho(v_{0\varphi}B_\theta - v_{0\theta}B_\varphi)\Bigg]\Bigg)B_\varphi + \\
&+ \frac{q}{m}\Bigg(\rho v_{0\varphi} - \tau\Bigg[\frac{\partial}{\partial t}(\rho v_{0\varphi}) + \frac{1}{r^2}\frac{\partial(r^2\rho v_{0r}v_{0\varphi})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\varphi}^2)}{\partial\varphi} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\theta}v_{0\varphi}\sin\theta)}{\partial\theta} + \\
&+ \frac{1}{r\sin\theta}\frac{\partial p}{\partial\varphi} - \rho g_\varphi - \frac{q}{m}\rho(v_{0\theta}B_r - v_{0r}B_\theta)\Bigg]\Bigg)B_r + \frac{1}{r^2}\frac{\partial}{\partial r}\Bigg\{r^2\Bigg\{\rho v_{0r}v_{0\theta} - \tau\Bigg[\frac{\partial}{\partial t}(\rho v_{0r}v_{0\theta}) + \\
&+ \frac{1}{r^2}\frac{\partial(r^2\rho v_{0r}^2v_{0\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\varphi}v_{0r}v_{0\theta})}{\partial\varphi} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\theta}^2v_{0r}\sin\theta)}{\partial\theta} - \\
&- g_r\rho v_{0\theta} - v_{0r}\rho g_\theta - \frac{q}{m}\rho(v_{0\varphi}B_\theta - v_{0\theta}B_\varphi)v_{0\theta} - v_{0r}\frac{q}{m}\rho(v_{0r}B_\varphi - v_{0\varphi}B_r)\Bigg]\Bigg\}\Bigg\} + \\
&+ \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\theta}^2v_{0\varphi}\sin\theta)}{\partial\theta} - g_\varphi\rho v_{0\theta} - \frac{q}{m}\rho(v_{0\theta}B_r - v_{0r}B_\theta)v_{0\theta} - v_{0\varphi}\frac{q}{m}\rho(v_{0r}B_\varphi - v_{0\varphi}B_r) - \\
&- v_{0\varphi}\rho g_\theta\Bigg]\Bigg\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\Bigg\{\sin\theta\Bigg\{\rho v_{0\theta}^2 - \tau\Bigg[\frac{\partial}{\partial t}(\rho v_{0\theta}^2) + \frac{1}{r^2}\frac{\partial(r^2\rho v_{0r}v_{0\theta}^2)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\varphi}v_{0\theta}^2)}{\partial\varphi} + \\
&+ \frac{1}{r\sin\theta}\frac{\partial(\rho v_{0\theta}^3\sin\theta)}{\partial\theta} - 2g_\theta\rho v_{0\theta} - \frac{q}{m}\rho(v_{0r}B_\varphi - v_{0\varphi}B_r)v_{0\theta}\Bigg]\Bigg\}\Bigg\} + \frac{1}{r}\frac{\partial p}{\partial\theta} - \frac{1}{r}\frac{\partial}{\partial\theta}\left(\tau\frac{\partial p}{\partial t}\right) - \\
&- \frac{2}{r}\frac{\partial}{\partial\theta}\left(\tau\left(\frac{1}{r^2}\frac{\partial(r^2pv_{0r})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(pv_{0\varphi})}{\partial\varphi} + \frac{1}{r\sin\theta}\frac{\partial(pv_{0\theta}\sin\theta)}{\partial\theta}\right)\right) - \\
&- \frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial(pv_{0\theta})}{\partial r}\right) - \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\tau\sin\theta\frac{\partial(pv_{0\theta})}{\partial\theta}\right) - \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(\tau\frac{\partial(pv_{0\theta})}{\partial\varphi}\right) = 0.
\end{aligned}
\tag{3.4}
$$

Energy equation:

$$\frac{\partial}{\partial t}\left\{\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{3}{2}p-\tau\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{3}{2}p\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_{0r}\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)\right)+\right.\right.$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(v_{0\varphi}\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta v_{0\theta}\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)\right)-$$

$$\left.\left.-\rho\left(g_r v_{0r}+g_\varphi v_{0\varphi}+g_\theta v_{0\theta}\right)\right]\right\}+$$

$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)v_{0r}-\tau\left[\frac{\partial}{\partial t}\left(\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)v_{0r}\right)+\right.\right.\right.$$

$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0r}^2\right)+$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0\varphi}v_{0r}\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0\theta}v_{0r}\right)-$$

$$-\rho\left(g_r v_{0r}+g_\varphi v_{0\varphi}+g_\theta v_{0\theta}\right)v_{0r}-\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{3}{2}p\right)g_r-$$

$$\left.\left.\left.-\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)\frac{q}{m}\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)\right]\right\}\right\}+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left\{\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)v_{0\varphi}-\right.$$

$$-\tau\left[\frac{\partial}{\partial t}\left(\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)v_{0\varphi}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0r}v_{0\varphi}\right)+\right.$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0\varphi}^2\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0\theta}v_{0\varphi}\right)-$$

$$-\rho\left(g_r v_{0r}+g_\varphi v_{0\varphi}+g_\theta v_{0\theta}\right)v_{0\varphi}-\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{3}{2}p\right)g_\varphi-$$

$$\left.\left.-\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)\frac{q}{m}\left(v_{0\theta}B_r-v_{0r}B_\theta\right)\right]\right\}+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\left\{\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)v_{0\theta}-\right.\right. \tag{3.5}$$

$$-\tau\left[\frac{\partial}{\partial t}\left(\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)v_{0\theta}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0r}v_{0\theta}\right)+\right.$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0\varphi}v_{0\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{7}{2}p\right)v_{0\theta}^2\right)-$$

$$\left.\left.\left.-\rho\left(g_r v_{0r}+g_\varphi v_{0\varphi}+g_\theta v_{0\theta}\right)v_{0\theta}-\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{3}{2}p\right)g_\theta-\left(\frac{1}{2}\rho v_0^2+\varepsilon n+\frac{5}{2}p\right)\frac{q}{m}\left(v_{0r}B_\varphi-v_{0\varphi}B_r\right)\right]\right\}\right\}-$$

$$-\left\{\rho\left(g_r v_{0r}+g_\varphi v_{0\varphi}+g_\theta v_{0\theta}\right)-\tau\left[g_r\left(\frac{\partial}{\partial t}(\rho v_{0r})+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v_{0r}^2\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\rho v_{0\varphi}v_{0r}\right)+\right.\right.\right.$$

$$\left.+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\rho v_{0\theta}v_{0r}\sin\theta\right)+\frac{\partial p}{\partial r}-\rho g_r-qn\left(v_{0\varphi}B_\theta-v_{0\theta}B_\varphi\right)\right)+$$

$$+g_\varphi\left(\frac{\partial}{\partial t}(\rho v_{0\varphi})+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v_{0r}v_{0\varphi}\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\rho v_{0\varphi}^2\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\rho v_{0\theta}v_{0\varphi}\sin\theta\right)+\right.$$

$$\left.+\frac{1}{r\sin\theta}\frac{\partial p}{\partial\varphi}-\rho g_\varphi-qn\left(v_{0\theta}B_r-v_{0r}B_\theta\right)\right)+g_\theta\left(\frac{\partial}{\partial t}(\rho v_{0\theta})+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v_{0r}v_{0\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\rho v_{0\varphi}v_{0\theta}\right)+\right.$$

$$\left.\left.\left.+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\rho v_{0\theta}^2\sin\theta\right)+\frac{1}{r}\frac{\partial p}{\partial\theta}-\rho g_\theta-qn\left(v_{0r}B_\varphi-v_{0\varphi}B_r\right)\right)\right]\right\}-$$

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial}{\partial r}\left(\frac{1}{2}pv_0^2+\varepsilon\frac{p}{m}+\frac{5}{2}\frac{p^2}{\rho}\right)\right)-$$

$$-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\tau\sin\theta\frac{\partial}{\partial\theta}\left(\frac{1}{2}pv_0^2+\varepsilon\frac{p}{m}+\frac{5}{2}\frac{p^2}{\rho}\right)\right)-\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(\tau\frac{\partial}{\partial\varphi}\left(\frac{1}{2}pv_0^2+\varepsilon\frac{p}{m}+\frac{5}{2}\frac{p^2}{\rho}\right)\right)+$$

$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau p g_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\tau p g_\varphi\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\tau p g_\theta\sin\theta\right)=0.$$

Let us point out the important particular non-stationary one dimensional case corresponding to the negative charged system evolution in the potential electric field:

(continuity equation)

$$\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}\right)}{\partial r}\right]\right\}+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_{0r}-\tau\left[\frac{\partial}{\partial t}(\rho v_{0r})+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2\right)}{\partial r}-q\frac{\partial\psi}{\partial r}\right]\right\}\right\}-$$
$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial p}{\partial r}\right)=0, \quad (3.6)$$

(momentum equation)

$$\frac{\partial}{\partial t}\left\{\rho v_{0r}-\tau\left[\frac{\partial}{\partial t}(\rho v_{0r})+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2\right)}{\partial r}+\frac{\partial p}{\partial r}-q\frac{\partial\psi}{\partial r}\right]\right\}-\frac{q}{\rho}\frac{\partial\psi}{\partial r}\left[\rho-\tau\left(\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}\right)}{\partial r}\right)\right]+$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_{0r}^2-\tau\left[\frac{\partial}{\partial t}\left(\rho v_{0r}^2\right)+\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^3\right)}{\partial r}-2q\frac{\partial\psi}{\partial r}v_{0r}\right]\right\}\right\}+ \quad (3.7)$$
$$+\frac{\partial p}{\partial r}-\frac{\partial}{\partial r}\left(\tau\frac{\partial p}{\partial t}\right)-2\frac{\partial}{\partial r}\left(\frac{\tau}{r^2}\frac{\partial\left(r^2 p v_{0r}\right)}{\partial r}\right)-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial\left(p v_{0r}\right)}{\partial r}\right)=0\,.$$

(energy equation)

$$\frac{\partial}{\partial t}\left\{\frac{1}{2}\rho v_{0r}^2+\frac{3}{2}p-\tau\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_{0r}^2+\frac{3}{2}p\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_{0r}\left(\frac{1}{2}\rho v_{0r}^2+\frac{5}{2}p\right)\right)-q\frac{\partial\psi}{\partial r}v_{0r}\right]\right\}+$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\left(\frac{1}{2}\rho v_{0r}^2+\frac{5}{2}p\right)v_{0r}-\tau\left[\frac{\partial}{\partial t}\left(\left(\frac{1}{2}\rho v_{0r}^2+\frac{5}{2}p\right)v_{0r}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{1}{2}\rho v_{0r}^2+\frac{7}{2}p\right)v_{0r}^2\right)-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.-q\frac{\partial\psi}{\partial r}v_{0r}^2-\frac{q}{\rho}\frac{\partial\psi}{\partial r}\left(\frac{1}{2}\rho v_{0r}^2+\frac{3}{2}p\right)\right)\right]\right\}\right\}-q\frac{\partial\psi}{\partial r}v_{0r}+ \quad (3.8)$$
$$+\tau\left[\frac{q}{\rho}\frac{\partial\psi}{\partial r}\left(\frac{\partial}{\partial t}(\rho v_{0r})+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v_{0r}^2\right)+\frac{\partial p}{\partial r}-q\frac{\partial\psi}{\partial r}\right)\right]-$$
$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial}{\partial r}\left(\frac{1}{2}p v_{0r}^2+\frac{5}{2}\frac{p^2}{\rho}\right)\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau p\frac{q}{\rho}\frac{\partial\psi}{\partial r}\right)=0\,.$$

Assume that non-stationary physical system is at the rest, namely $v_{0r}=0$. Taking into account also the forces of the magnetic origin one obtains from the system of equations (3.1) – (3.5) for the non-stationary one-dimensional (along r) case:
(continuity equation)

$$\frac{\partial}{\partial t}\left\{\rho-\tau\frac{\partial\rho}{\partial t}\right\}+\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\tau\left(q\frac{\partial\psi}{\partial r}-\frac{\partial p}{\partial r}\right)\right]=0, \quad (3.9)$$

(momentum equation, $\mathbf{e}_r$ projection)

$$\frac{\partial}{\partial t}\left[\tau q\frac{\partial\psi}{\partial r}\right]-\frac{q}{\rho}\frac{\partial\psi}{\partial r}\left[\rho-\tau\frac{\partial\rho}{\partial t}\right]+$$
$$+\frac{\partial}{\partial r}\left[p-\tau\frac{\partial p}{\partial t}\right]=0\,, \quad (3.10)$$

(momentum equation, $\mathbf{e}_\varphi$ projection)

$$\frac{q}{m}\tau\left[\frac{\partial p}{\partial r}-q\frac{\partial\psi}{\partial r}\right]B_\theta=0\,, \quad (3.11)$$

(momentum equation, $\mathbf{e}_\theta$ projection):

$$\tau\frac{q}{m}\left[\frac{\partial p}{\partial r}-q\frac{\partial\psi}{\partial r}\right]B_\varphi=0. \quad (3.12)$$

(energy equation)

$$\frac{\partial}{\partial t}\left\{\varepsilon n+\frac{3}{2}p-\tau\frac{\partial}{\partial t}\left(\varepsilon n+\frac{3}{2}p\right)\right\}=$$
$$=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial}{\partial r}\left[\frac{p}{\rho}\left(\varepsilon n+\frac{5}{2}p\right)\right]\right)-$$
$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left\{\tau r^2\left(\varepsilon n+\frac{5}{2}p\right)\frac{q}{\rho}\frac{\partial\psi}{\partial r}\right\}- \quad (3.13)$$
$$-\tau\frac{q}{\rho}\frac{\partial\psi}{\partial r}\left(\frac{\partial p}{\partial r}-q\frac{\partial\psi}{\partial r}\right),$$

where ε is the internal particle energy. To the system of equations (3.9), (3.10), (3.13) the Poisson equation should be added:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)=4\pi q\,, \quad (3.14)$$

where ψ – scalar electric potential and q is the absolute value of the negative charge (per the unit of volume) of the one species quantum object.

4. The derivation of the angle relaxation equation

Let us consider an electron which is at rest at the time moment $t = 0$. This electron has the internal energy ε (see also (2.20), (2.21))

$$\varepsilon = \varepsilon_{el,sp} + \varepsilon_{el,m}, \tag{4.1}$$

containing the spin and magnetic parts, namely

$$\varepsilon_{el,sp} = \hbar\omega/2, \ \ \varepsilon_{el,m} = -\mathbf{p_m}\cdot\mathbf{B}, \tag{4.2}$$

$\mathbf{p_m}$ – electron magnetic moment, $\mathbf{B}$ – magnetic induction. But $p_m = -\frac{e}{m_e}\frac{\hbar}{2c}$ and relation (4.1) can be written as:

$$\varepsilon = \frac{\hbar}{2}\left[\omega + \frac{e}{m_e c}B\cos\vartheta\right], \tag{4.3}$$

where the angle ϑ reflects the possible deviation between a separated direction of the spin at the initial time moment and the direction of magnetic momentum after an external perturbation. For example this perturbation can be considered as result of the approach of the second electron to the previous one at the distance r_{in} with appearance of the virtual photon with the wavelength:

$$\lambda_{ph} = 2\pi r_{in}. \tag{4.4}$$

The fine-structure constant α has the physical interpretations as the ratio of two energies:

(i) the energy E_c needed to overcome the electrostatic repulsion between two electrons a distance of r_{in} apart, and

(ii) the energy of a single photon of wavelength $\lambda_{ph} = 2\pi r_{in}$.

Taking into account the previous remarks let us consider the charge time evolution inside of the first electron. In principle we need to solve the general complicated system (3.1)–(3.5). It is reasonable to obtain much more simple solution using the perturbation method. Namely, all unknown functions can be expanded in a Taylor series like:

$$\rho = \rho_0 + \left[\frac{\partial\rho}{\partial t}\right]_{t=t_0}\delta t + ... \tag{4.5}$$

In particular we need to find the time derivation of the value $\varepsilon_{el,m} = -\mathbf{p_m}\cdot\mathbf{B}$ and therefore the derivative with signs reflecting two possible projection orientations $\pm\frac{\partial}{\partial t}\cos\vartheta = \mp\sin\vartheta\frac{\partial\vartheta}{\partial t}$. The derivative $\frac{\partial\vartheta}{\partial t}$ is written in the relaxation form:

$$\frac{\partial\vartheta}{\partial t} = \frac{\pi}{\tau}. \tag{4.6}$$

As it was supposed the deviation of the magnetic moment from the spin orientation is result of the approach of the second electron with impulse p to the first electron at the distance r_{in}. In this case:

$$\frac{1}{\tau} = \frac{p}{m_e\lambda}, \tag{4.7}$$

where $\lambda \sim r_{in}$. After introduction the coefficient s, we have $r_{in} = \lambda s$ and

$$\frac{1}{\tau} = s\frac{p}{m_e r_{in}}. \tag{4.8}$$

It means

$$\frac{\partial\vartheta}{\partial t} = s\frac{\pi p}{m_e r_{in}}. \tag{4.9}$$

or

$$\frac{\partial\vartheta}{\partial t} = s\frac{2\pi}{p r_{in}}\frac{p^2}{2m_e} = s\frac{2\pi}{p r_{in}}E_c. \tag{4.10}$$

Let us introduce now the fine-structure constant α

$$\alpha = \frac{E_c}{E_{ph}}. \tag{4.11}$$

and transform (4.10)

$$\frac{\partial\vartheta}{\partial t} = \frac{2\pi}{p r_{in}}s\alpha E_{ph}, \tag{4.12}$$

$$\frac{\partial\vartheta}{\partial t} = \frac{2\pi}{h}s\alpha E_{ph}\frac{\lambda}{r_{in}}. \tag{4.13}$$

and using $r_{in} = \lambda s$ one obtains

$$\pm\sin\vartheta\frac{\partial\vartheta}{\partial t} = \pm\frac{2\pi}{h}\alpha E_{ph}\sin\vartheta = \pm\alpha\omega_{in}\sin\vartheta. \tag{4.14}$$

or

$$\frac{\partial\vartheta}{\partial t} = \alpha\omega_{in}, \tag{4.15}$$

where ω_{in} is the photon frequency which the wave length is $2\pi r_{in}$. But

$$\alpha = \frac{e^2}{\hbar c}. \tag{4.16}$$

It means, that equation (4.15) takes the transparent physical form

$$\frac{\partial\vartheta}{\partial t} = \frac{e^2}{\hbar r_{in}}. \tag{4.17}$$

5. The mathematical modeling of the charge distribution in electron

Let us deliver the derivation of the non-local equations in the first approximation. From (3.9)–(3.12) follows

$$\frac{\partial p}{\partial r} - q\frac{\partial\psi}{\partial r} = 0. \tag{5.1}$$

Transform the energy equation (3.13) using (4.14), (5.1), (see also (2.22))

$$\pm\frac{\hbar}{2}n\frac{e}{m_e c}B\alpha\omega_{in}\sin\vartheta =$$

$$=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial}{\partial r}\left[\frac{p}{\rho}\left(\varepsilon n+\frac{5}{2}p\right)\right]\right)- \quad (5.2)$$

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left\{\tau r^2\left(\varepsilon n+\frac{5}{2}p\right)\frac{q}{\rho}\frac{\partial\psi}{\partial r}\right\}.$$

or

$$\pm\frac{\hbar}{2}n\frac{e}{m_e c}B\alpha\omega_{in}r^2\sin\vartheta=\frac{\partial}{\partial r}\left(\tau r^2\frac{p}{m}\frac{\partial\varepsilon}{\partial r}\right)+ \quad (5.3)$$

$$+\frac{5}{2}\frac{\partial}{\partial r}\left(\tau r^2 p\frac{\partial}{\partial r}\left[\frac{p}{\rho}\right]\right).$$

Naturally to suppose that $\frac{\partial\varepsilon}{\partial r}$=0 and non-local parameter τ does not depend on r, then:

$$\pm\frac{\hbar}{2c}nB\alpha\omega_{in}r^2\sin\vartheta=\frac{5}{2}\tau\frac{\partial}{\partial r}\left(r^2 p\frac{\partial}{\partial r}\left[\frac{p}{q}\right]\right). \quad (5.4)$$

Using the relation $\rho = m\frac{q}{e}$, scales r_0, p_0, q_0 for the values r, p, q and denoting by tilde the dimensionless values one obtains

$$\pm\frac{\hbar}{5c}\frac{1}{e}B\frac{\omega_{in}}{\tau}\frac{r_0^2 q_0^2}{p_0^2}\alpha\sin\vartheta\;\tilde r^2\tilde q=\frac{\partial}{\partial\tilde r}\left(\tilde r^2\tilde p\frac{\partial}{\partial\tilde r}\left[\frac{\tilde p}{\tilde q}\right]\right). \quad (5.5)$$

Introduce the notation $\tilde B$ for the dimensionless coefficient

$$\tilde B=\frac{\hbar}{5c}\frac{1}{e}B\frac{\omega_{in}}{\tau}\frac{r_0^2 q_0^2}{p_0^2}\alpha\sin\vartheta, \quad (5.6)$$

we have

$$\frac{\partial}{\partial\tilde r}\left(\tilde r^2\tilde p\frac{\partial}{\partial\tilde r}\left[\frac{\tilde p}{\tilde q}\right]\right)=\pm\tilde B\tilde r^2\tilde q. \quad (5.7)$$

The Poisson equation (3.14) takes the dimensionless form

$$A\frac{\partial}{\partial\tilde r}\left(\tilde r^2\frac{\partial\tilde\psi}{\partial\tilde r}\right)=\tilde r^2\tilde q, \quad (5.8)$$

where the dimensionless coefficient A is introduced

$$A=\frac{\psi_0}{4\pi r_0^2 q_0}, \quad (5.9)$$

ψ_0 is the scale for the potential ψ.

In the absence of perturbations $\tilde B = 0$ and from (5.7) one obtains

$$p = Cq. \quad (5.10)$$

From (5.7), (5.8) follow also

$$\frac{\partial}{\partial\tilde r}\left(\tilde r^2\tilde p\frac{\partial}{\partial\tilde r}\left[\frac{\tilde p}{\tilde q}\right]\right)=\pm\tilde B A\frac{\partial}{\partial\tilde r}\left(\tilde r^2\frac{\partial\tilde\psi}{\partial\tilde r}\right). \quad (5.11)$$

Write down the equation (5.1) in the dimensionless form

$$\frac{p_0}{q_0\psi_0}\frac{\partial\tilde p}{\partial\tilde r}-\tilde q\frac{\partial\tilde\psi}{\partial\tilde r}=0. \quad (5.12)$$

and introduce the obvious relation between scales for the simplification

$$\frac{p_0}{q_0\psi_0}=1, \quad (5.13)$$

then

$$\frac{\partial\tilde p}{\partial\tilde r}-\tilde q\frac{\partial\tilde\psi}{\partial\tilde r}=0. \quad (5.14)$$

and

$$\tilde B = B\frac{\hbar\omega_{in}}{\tau c}\frac{r_0^2}{5e\psi_0^2}\alpha\sin\vartheta. \quad (5.15)$$

Using equations (5.11), (5.14) it turns out that

$$\frac{\partial\tilde p}{\partial\tilde r}\pm\tilde B A\frac{\partial\ln\tilde p}{\partial\tilde r}=\frac{\tilde p}{\tilde q}\frac{\partial\tilde q}{\partial\tilde r}, \quad (5.16)$$

$$\frac{\partial}{\partial\tilde r}\left(\ln\frac{\tilde p}{\tilde q}\right)\pm\tilde B A\frac{1}{p}\frac{\partial\ln\tilde p}{\partial\tilde r}=0, \quad (5.17)$$

then the second term on the left hand side of Eq. (5.16) reflects the influence of perturbation. Omitting this term we return to the relation (5.10).

Before going further some points need to be made about so called the “classical electron radius”. This is a calculated radius based on an assumption that the electron is the empty charged sphere a certain radius. It has a value of r_0 = $2.82\cdot10^{-15}$ m obtained as result of calculation by equating the potential electrostatic energy e^2/r_0 to the energy of rest $m_e c^2$. Now compare this radius with the measured radius of a proton, which is $1.11\cdot10^{-15}$ m. There are several sources with different values, but they appear to be around 10^{-15} m. According to this an electron has a radius 2.5 times larger than a proton. Given that a proton is 1836 heavier however, it’s difficult to know if we should take this “classical radius” seriously.

Write down once more the system of equation which was used in the mathematical modeling (SYSTEM I)

$$\frac{\partial}{\partial\tilde r}\left(\tilde r^2\tilde p\frac{\partial}{\partial\tilde r}\left[\frac{\tilde p}{\tilde q}\right]\right)=\tilde B\tilde r^2\tilde q,$$

$$A\frac{\partial}{\partial\tilde r}\left(\tilde r^2\frac{\partial\tilde\psi}{\partial\tilde r}\right)=\tilde r^2\tilde q,$$

$$\tilde q\frac{\partial\tilde\psi}{\partial\tilde r}-\frac{\partial\tilde p}{\partial\tilde r}=0,$$

where

$$A=\frac{\psi_0}{4\pi r_0^2 q_0},\quad \tilde B=\pm B\frac{\hbar\omega_{in}}{\tau c}\frac{r_0^2}{5e\psi_0^2}\alpha\sin\vartheta.$$

Some significant remarks:

1. Solutions of SYSTEM I belongs to the class of Cauchy problems and need not in introduction the strictly defined the electron radius beforehand.

2. From here on for convenience the different signs were included in $\tilde{B}$.

3. The mentioned classical radius r_0 is only one from possible scales.

Really, from (5.8) follows that the absolute electron charge q_{el} is equal to

$$q_{el} = |e| = \int_0^{r_{el}} 4\pi r^2 q(r)dr = 4\pi r_0^3 q_0 \int_0^{\tilde{r}_{el}} \tilde{r}^2 \tilde{q} d\tilde{r} =$$
$$= 4\pi r_0^3 q_0 A \int_0^{\tilde{r}_{el}} \frac{\partial}{\partial \tilde{r}}\left(\tilde{r}^2 \frac{\partial \tilde{\psi}}{\partial \tilde{r}}\right) d\tilde{r} =$$
$$= r_0 \psi_0 \int_0^{\tilde{r}_{el}} \frac{\partial}{\partial \tilde{r}}\left(\tilde{r}^2 \frac{\partial \tilde{\psi}}{\partial \tilde{r}}\right) d\tilde{r} = \tag{5.18}$$
$$= r_0 \psi_0 \tilde{r}_{el}^2 \left[\frac{\partial \tilde{\psi}}{\partial \tilde{r}}\right]_{r=r_{el}},$$

or

$$\left[\frac{\partial \tilde{\psi}}{\partial \tilde{r}}\right]_{r=r_{el}} = 1. \tag{5.19}$$

for the scales choosed as

$$\psi_0 = |e|/r_0,\ r_0 = r_{el},\ q_0 = |e|/r_{el}^3,$$
$$p_0 = q_0\psi_0 = \frac{e^2}{r_{el}^4}. \tag{5.20}$$

In this case

$$\left[\frac{\partial \psi}{\partial r}\right]_{r=r_{el}} = \frac{|e|}{r_0^2}, \tag{5.21}$$

or

$$[F]_{r=r_{el}} = \frac{e^2}{r_{el}^2}. \tag{5.22}$$

But in the definition of the fine-structure constant α the energy E_c was introduced as the energy needed to overcome the electrostatic repulsion between two electrons a distance of r_{in} apart (see also (4.11)). It means that for this problem naturally to put the scale $r_0 = r_{in}$. In this case (system of conditions SYSTEM II):

$$\psi_0 = |e|/r_0,\ r_0 = r_{in},\ q_0 = |e|/r_{in}^3,$$
$$p_0 = q_0\psi_0 = e^2 / r_{in}^4.$$
$$A = \frac{\psi_0}{4\pi r_0^2 q_0} = \frac{1}{4\pi},$$
$$\tilde{B} = \pm B \frac{\hbar\omega_{in}}{\tau c} \frac{r_{in}^4}{5|e|^3} \alpha \sin\vartheta.$$

Parameter (5.15) can be rewritten as

$$\tilde{B} = \pm B \frac{\hbar\omega_{in}}{\tau c} \frac{r_{in}^4}{5|e|^3} \alpha \sin\vartheta = \pm B \frac{r_{in}^3}{5|e|\tau c} \sin\vartheta. \tag{5.23}$$

Let us introduce the character magnetic force

$$F_{mag} = \frac{|e|}{c} \frac{r_{in}}{5\tau} B. \tag{5.24}$$

and the character electrostatic force

$$F_{elect} = \frac{e^2}{r_{in}^2}. \tag{5.25}$$

It means that parameter $\tilde{B}$ can be written in the transparent physical form

$$\tilde{B} = \frac{F_{mag}}{F_{elect}} \sin\vartheta. \tag{5.26}$$

Is it possible to obtain the soliton type solution for this object under these conditions? Let us show that the System I admits such kind of solutions.

All following calculations are realized under conditions SYSTEM II (in particular by $A = \frac{1}{4\pi}$, different $\tilde{B}$ and initial conditions). The influence $\tilde{B}$ is investigated from zero up to value $|\tilde{B}|=10$. Maple notations are used ($\mathrm{v} = \tilde{\psi}$, $\mathrm{D(v)(t)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}$, $\mathrm{q} = \tilde{q}$, $\mathrm{t} = \tilde{r}$, $\mathrm{B} = \tilde{B}$). Cauchy conditions for the calculations reflected on figures 3–20:

$$\mathrm{v}(0) = \tilde{\psi}(0) = 1,\ \mathrm{D(v)(0)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(0) = 0;$$
$$\mathrm{q}(0) = \tilde{q}(0) = 1,\ \mathrm{D(q)(0)} = \frac{\partial \tilde{q}}{\partial \tilde{r}}(0) = 0;$$
$$\mathrm{p}(0) = \tilde{p}(0) = 1,\ \mathrm{D(p)(0)} = \frac{\partial \tilde{p}}{\partial \tilde{r}}(0) = 0.$$

Figures 3–5 correspond to the case when the angle ϑ is nil and then $\tilde{B} = 0$. Solutions in all calculations exist only in a bounded region of the 1D space. The size of this region $r_{\lim}$ defines the electron radius. For the case $\tilde{B} = 0$ one obtains $\tilde{r}_{\lim} = 0.9235$.

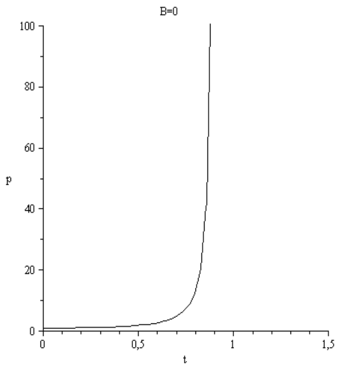

Fig. 3. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = 0$.

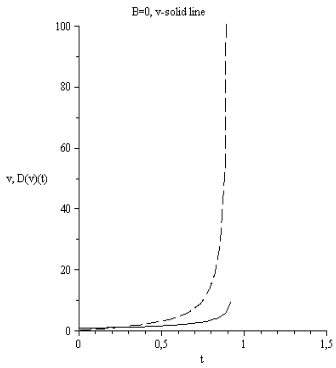

Fig. 4. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{D(v)(t)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\tilde{B} = 0$.

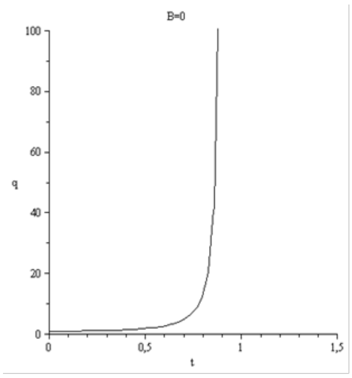

Fig. 5. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\tilde{B} = 0$.

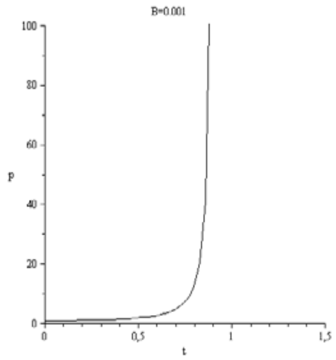

Fig. 6. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = 0.001$.

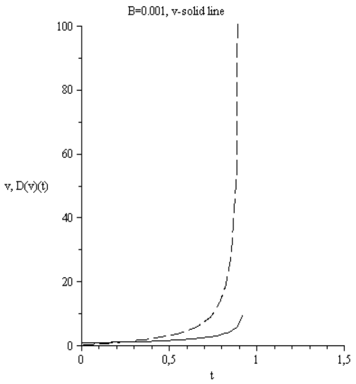

Fig. 7. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{D(v)(t)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\tilde{B} = 0.001$.

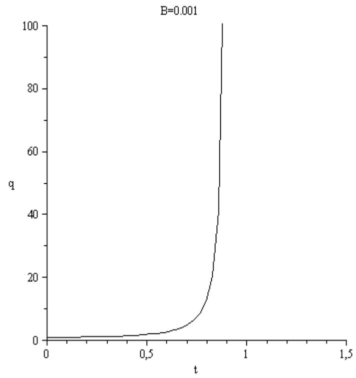

Fig. 8. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\tilde{B} = 0.001$.

For the case $\mathrm{B} = \tilde{B} = 0.001$ one obtains also $\tilde{r}_{\lim} = 0.9235$.

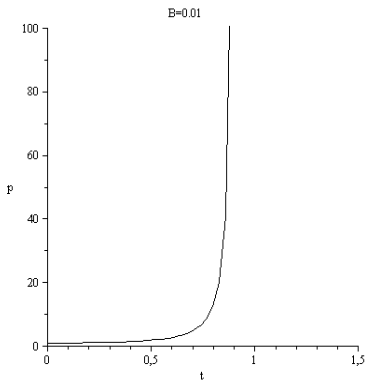

Fig. 9. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = 0.01$.

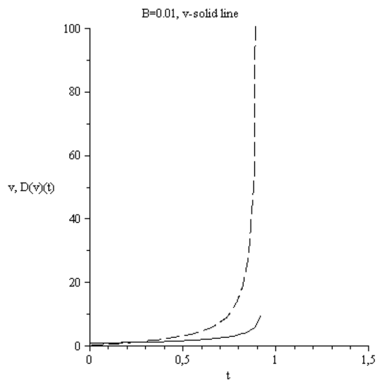

Fig. 10. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{D(v)(t)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\tilde{B} = 0.01$.

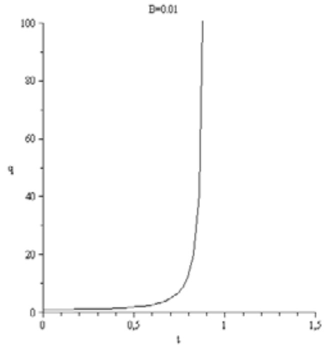

Fig. 11. $q = \tilde{q}(\tilde{r})$, $\tilde{B} = 0.01$.

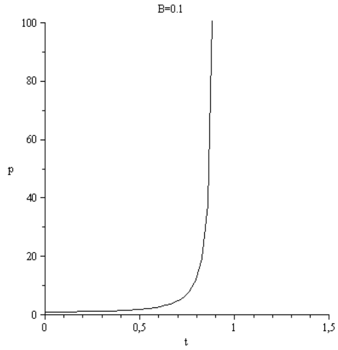

Fig. 12. $p = \tilde{p}(\tilde{r})$, $\tilde{B} = 0.1$.

For the case $B = \tilde{B} = 0.01$ one obtains $\tilde{r}_{\lim} = 0.9239$.

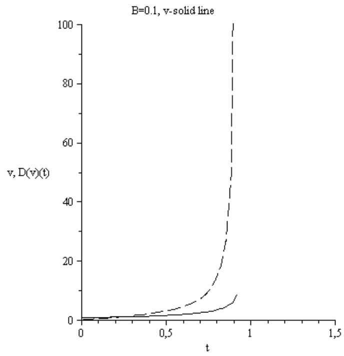

Fig. 13. $v = \tilde{\psi}(\tilde{r})$, $D(v)(t) = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $v = \tilde{\psi}(\tilde{r})$, $\tilde{B} = 0.1$.

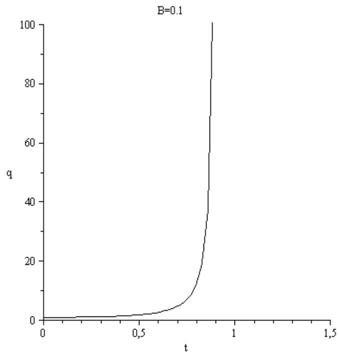

Fig. 14. $q = \tilde{q}(\tilde{r})$, $\tilde{B} = 0.1$.

For the case $B = \tilde{B} = 0.1$ one obtains $\tilde{r}_{\lim} = 0.9272$.

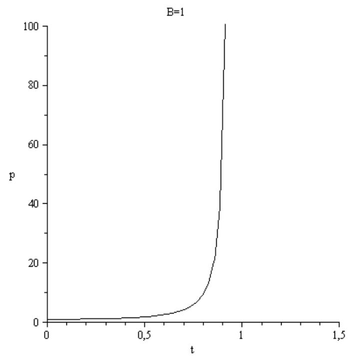

Fig. 15. $p = \tilde{p}(\tilde{r})$, $\tilde{B} = 1$.

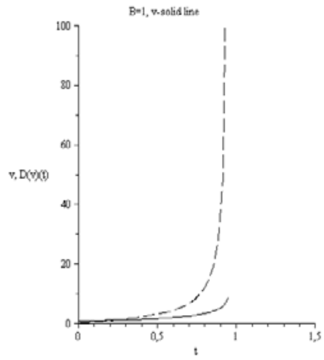

Fig. 16. $v = \tilde{\psi}(\tilde{r})$, $D(v)(t) = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $v = \tilde{\psi}(\tilde{r})$, $\tilde{B} = 1$.

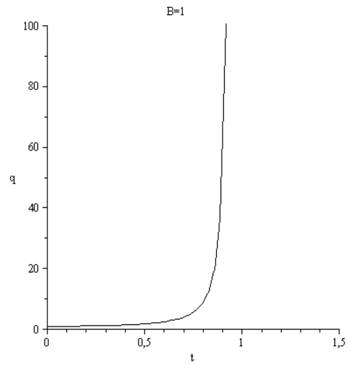

Fig. 17. $q = \tilde{q}(\tilde{r})$, $\tilde{B} = 1$.

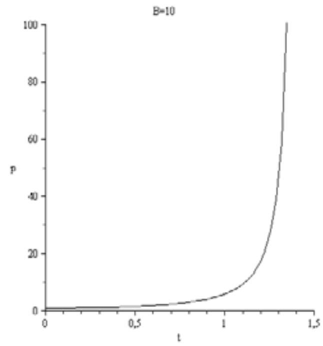

Fig. 18. $p = \tilde{p}(\tilde{r})$, $\tilde{B} = 10$.

For the case $B = \tilde{B} = 1$ one obtains $\tilde{r}_{\lim} = 0.9614$.

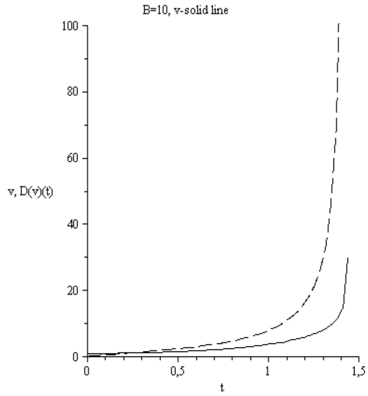

Fig. 19. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{D(v)(t)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\tilde{B} = 10$.

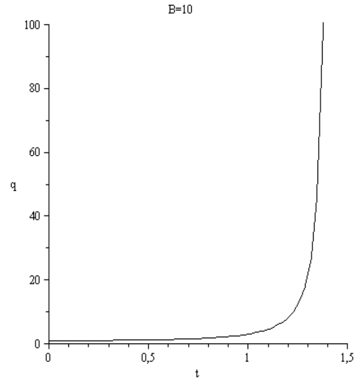

Fig. 20. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\tilde{B} = 10$.

For the case $\mathrm{B} = \tilde{B} = 10$ one obtains $\tilde{r}_{\lim} = 1.4397$. Calculations reflected on figures 21 – 23 are realized by conditions SYSTEM III: $\mathrm{B} = \tilde{B} = 0.1$, $\mathrm{v}(0) = \tilde{\psi}(0) = 1$, $\mathrm{D(v)(0)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(0) = 0$; $\mathrm{q}(0) = \tilde{q}(0) = 0.1$, $\mathrm{D(q)(0)} = \frac{\partial \tilde{q}}{\partial \tilde{r}}(0) = 0$, $p(0) = \tilde{p}(0) = 0.01$, $\mathrm{D(p)(0)} = \frac{\partial \tilde{p}}{\partial \tilde{r}}(0) = 0$.

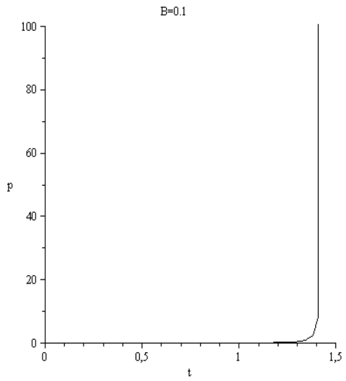

Fig. 21. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = 0.1$.

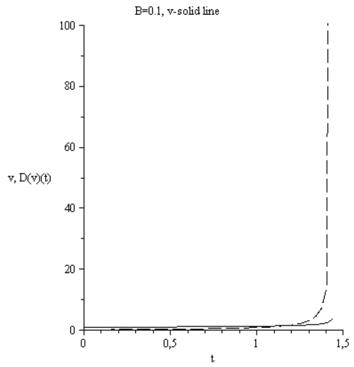

Fig. 22. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{D(v)(t)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\tilde{B} = 0.1$.

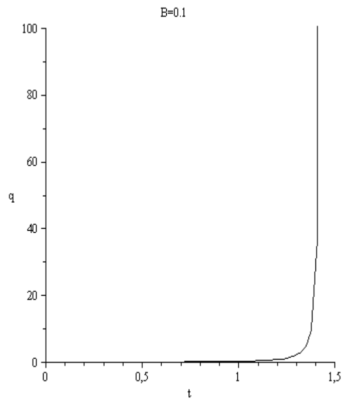

Fig. 23. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\tilde{B} = 0.1$.

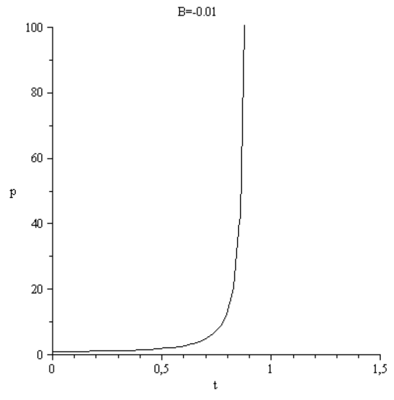

Fig. 24. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = -0.01$.

For the case SYSTEM III one obtains $\tilde{r}_{\lim} = 1.44$.

Figures 24 – 38 demonstrate the results of calculations for the negative values $B = \tilde{\mathrm{B}}$ but for the Cauchy conditions:

$$\mathrm{v}(0) = \tilde{\psi}(0) = 1, \quad \mathrm{D(v)(0)} = \frac{\partial \tilde{\psi}}{\partial \tilde{r}}(0) = 0; \; \mathrm{q}(0) = \tilde{q}(0) = 1, \; \mathrm{D(q)(0)} = \frac{\partial \tilde{q}}{\partial \tilde{r}}(0) = 0, \; p(0) = \tilde{p}(0) = 1,$$

$$\mathrm{D(p)(0)} = \frac{\partial \tilde{p}}{\partial \tilde{r}}(0) = 0.$$

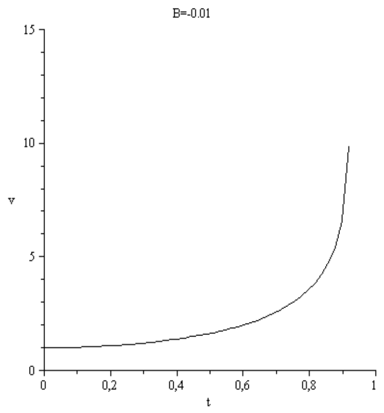

Fig. 25. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.01$.

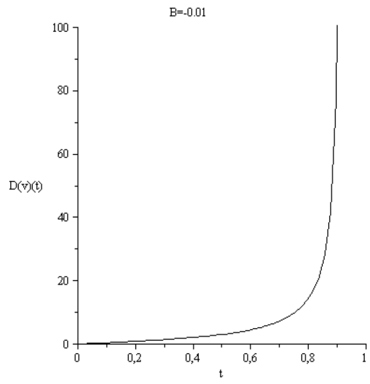

Fig. 26. $\mathrm{D(v)(t)} = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.01$.

For the case $\mathrm{B} = \tilde{B} = -0.01$ one obtains $\tilde{r}_{\lim} = 0.92312$.

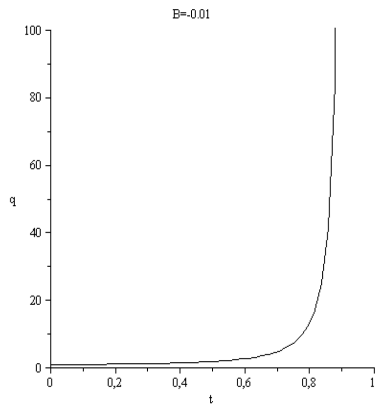

Fig. 27. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.01$.

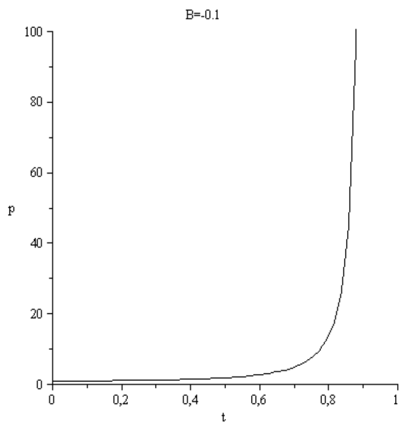

Fig. 28. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = -0.1$.

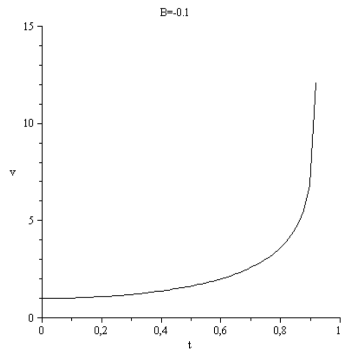

Fig. 29. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.1$.

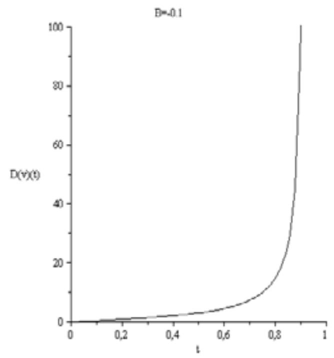

Fig. 30. $\mathrm{D(v)(t)} = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.1$.

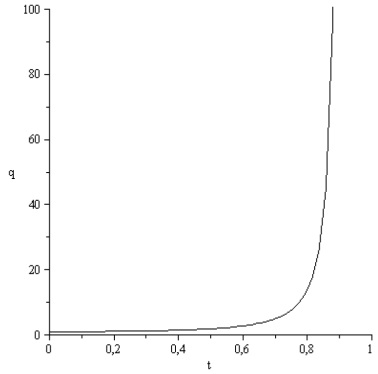

Fig. 31. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.1$.

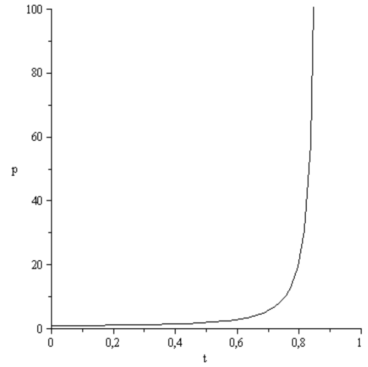

Fig. 32. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = -1$.

For the case $\mathrm{B} = \tilde{B} = -0.1$ one obtains $\tilde{r}_{\lim} = 0.9198$.

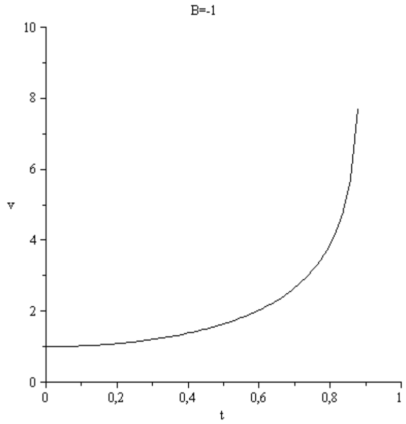

Fig. 33. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -1$.

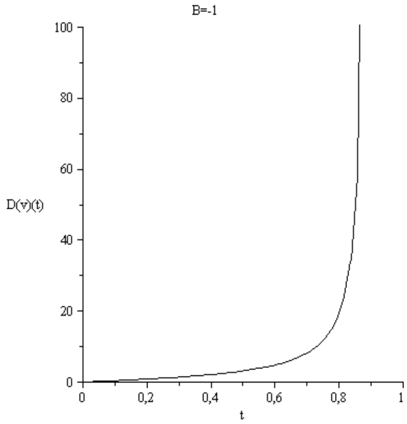

Fig. 34. $\mathrm{D(v)(t)} = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -1$.

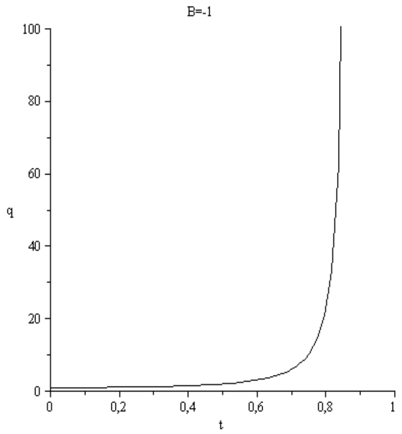

Fig. 35. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -1$.

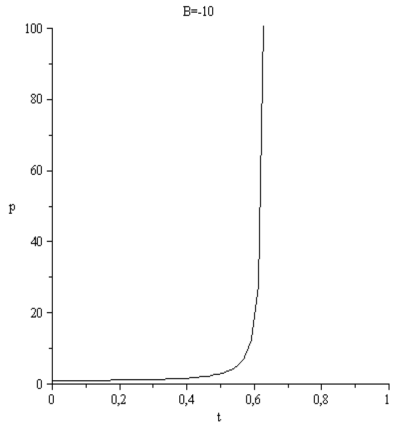

Fig. 36. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\tilde{B} = -10$.

For the case $\mathrm{B} = \tilde{B} = -1$ one obtains $\tilde{r}_{\lim} = 0.8979$.

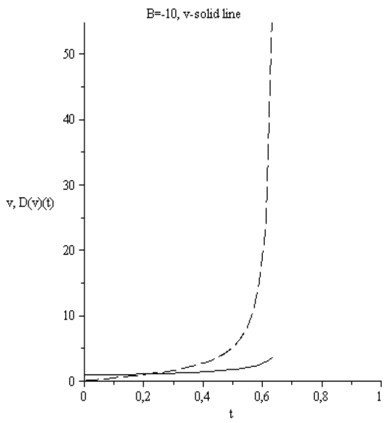

Fig. 37. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{D(v)(t)} = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$,

solid line $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -10$.

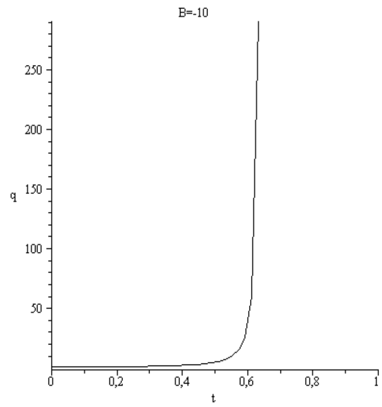

Fig. 38. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -10$.

For the case $\mathrm{B} = \tilde{B} = -10$ one obtains $\tilde{r}_{\lim} = 0.6487$. Finally I show some results obtained for the case $\mathrm{v}(0) = \tilde{\psi}(0) = 1$, $\mathrm{D(v)(0)} = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(0) = 0$; $\mathrm{q}(0) = \tilde{q}(0) = 0.1$, $\mathrm{D(q)(0)} = \dfrac{\partial \tilde{q}}{\partial \tilde{r}}(0) = 0$, $p(0) = \tilde{p}(0) = 0.01$, $\mathrm{D(p)(0)} = \dfrac{\partial \tilde{p}}{\partial \tilde{r}}(0) = 0$ but for the negative value $B = \tilde{\mathrm{B}} = -0.1$; compare fig. 39–41 with fig. 21–23.

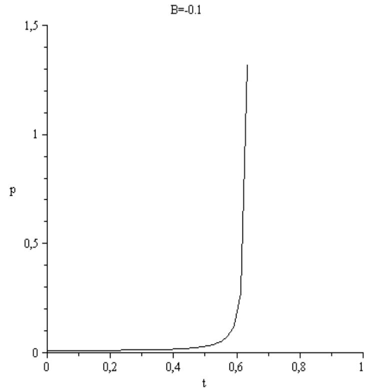

Fig. 39. $\mathrm{p} = \tilde{p}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.1$.

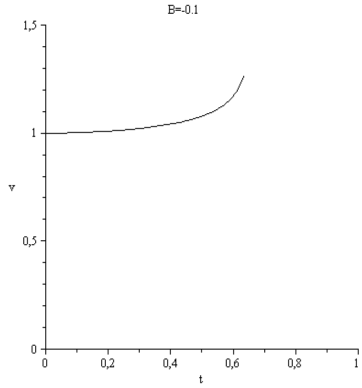

Fig. 40. $\mathrm{v} = \tilde{\psi}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.1$.

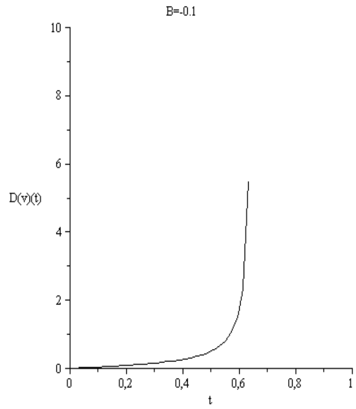

Fig. 41. $\mathrm{D(v)(t)} = \dfrac{\partial \tilde{\psi}}{\partial \tilde{r}}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.1$.

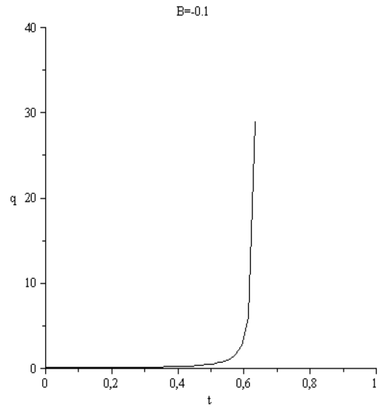

Fig. 42. $\mathrm{q} = \tilde{q}(\tilde{r})$, $\mathrm{B} = \tilde{B} = -0.1$.

For the last case $\mathrm{B} = \tilde{B} = -0.1$ one obtains $\tilde{r}_{\lim} = 0.6487$.

Some conclusions from delivered calculations:

1. From calculations follow that electrons can be considered like charged balls (shortly CB model) which charges are concentrated mainly in the shell of these balls. In the first approximation (when $\vartheta = 0$) this result does not depend on the choice of the non-locality parameter.

2. Electron radius can not be indicated exactly in principle; its radius depends on physical system where an electron is placed. It is possible to speak about the different electron shells connected with evolution of the charge density, quantum pressure, electric potential and forces near the boundary.

3. From the theoretical point of view the electron size is the size of domain of the existence of the corresponding solution. The mentioned sizes $\tilde{r}_{\lim}$ are indicated for all considered cases; the values $\tilde{r}_{\lim}$ practically do not depend on the chosen numerical method.

4. The value of $\tilde{r}_{\lim}$ depends significantly on choosing of the Cauchy conditions. By the same Cauchy conditions the weak dependence on parameter $\tilde{B}$ exists only for the moderate value of this parameter. If $|\tilde{B}|$ is of the unit order or more the value $\tilde{r}_{\lim}$ may vary very significantly especially with changing of sign in front of $\tilde{B}$.

5. The proton-electron collision in the frame of CB-model should be considered as collision of two resonators. Curves of the equal amplitudes of the intensity of electric field create domains in proton in the form of many “islands” – caustic surfaces of electromagnetic field which can serve as additional scattering centers. It can open new way for explanation a number of character collisional features depending on the initial and final electron energies without consideration partons or quarks as scattering centers, [11].

6. This results should be taken into account in the theory of the single floating electron been isolated in a Penning trap (see for example [22, 23]).

In this connection another interesting problem is arising. Can be experimentally confirmed the resonator model for the electron? In this case it is reasonable to remind one old Blokhintsev paper published in Physics-Uspekhi as the letter to Editor [24]. He considered the process of the interaction neutrino υ and electron e with transformation of electron in μ – meson $\upsilon + \mathrm{e} \to \mu + \upsilon'$. In this case the energy density W can be estimated as

$$W = g^{*}\overline{\psi}_{\mathrm{e}}\psi_{\mu}\overline{\psi}_{\upsilon}\psi_{\upsilon'} \,, \qquad (5.27)$$

where g^* is Fermi constant, ψ_e, ψ_μ, ψ_υ are wave functions for electron, μ – meson and neutrino correspondingly. Following I.S. Shapiro, Blokhintsev estimated g^* as

$$g^*/(\hbar c) = \Lambda_0^2\,, \qquad (5.28)$$

with $\Lambda_0 \sim 10^{-16}\,cm$. His conclusion consists in affirmation that the strong interaction of electron and neutrino takes place when the wave length $\lambda\!\!\!^{-}$ of the neutrino wave packet less than Λ_0.

$$\lambda\!\!\!^{-} < \Lambda_0\,. \qquad (5.29)$$

The inequality (5.29) can be considered as estimation for revealing of the resonance electron properties. Blohintzev supposes that fulfilling of (5.29) leads to the significant changes in the Compton effect and to other changes in electro-magnetic interaction of electrons. It is possible also to wait for the influence of the resonance electron effects on investigation of hypothetical neutrino oscillations.

6. Conclusion

The origin of the charge density and spin waves is a long-standing problem relevant to a number of important issues in condensed matter physics. The collective excitations are discussed here in view of quantum non-local hydrodynamics. Whereas the latter remains valid in graphene and yields insight into the understanding of spin – charge dependent modes, the generalized system of equations is derived including possible particular cases. It is known that the Schrödinger – Madelung quantum physics leads to the destruction of the wave packets and can not be used for the solution of this kind of problems. The appearance of the soliton solutions in mathematics is the rare and remarkable effect. As we see the soliton's appearance in the generalized hydrodynamics created by Alexeev is an "ordinary" oft-recurring fact. Investigation of the inner charge distribution of electron in the frame of the non-local quantum hydrodynamics leads to following main results:

1. From calculations follow that electron can be considered like charged ball (shortly CB model) which charge is concentrated mainly in the shell of this ball. In the first approximation this result does not depend on the choice of the non-locality parameter.
2. Electron radius can not be indicated exactly in principle; its radius depends on physical system where an electron is placed. It is possible to speak about the different electron shells connected with evolution of the charge density, quantum pressure, electric potential and forces near the boundary.
3. These results should be taken into account in the theory of the single floating electron been isolated in a Penning trap.

Important to underline that the problem of existing and propagation of solitons belongs to the class of significantly non-local non-linear problems which can be solved only in the frame of vast numerical modeling.

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К НЕЛОКАЛЬНОЙ ТЕОРИИ ЗАРЯДОВЫХ И СПИНОВЫХ ВЗАИМОДЕЙСТВИЙ В ВОЛНАХ И ЧАСТИЦАХ

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В рамках нелокальной квантовой гидродинамики построена теория взаимодействия в волнах зарядовых и спиновых возбуждений. Исследована внутренняя зарядовая структура электрона на основе нелокального описания. Из расчетов следует, что внутреннее распределение заряда электрона отвечает модели шара, заряд которого сосредоточен в основном в окрестности оболочки шара. В расчетах учитывается возможное отклонение спина от направления магнитного момента.

Ключевые слова: *основы теории процессов переноса, теория солитонов, обобщенные гидродинамические уравнения, основания квантовой механики.*