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TO THE NON-LOCAL THEORY OF LEVITATION

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In this paper we consider the levitation phenomenon using the generalized Bolzmann kinetics theory which can represent the non-local physics of levitation. This approach can identify the conditions when the levitation can take place under the influence of correlated electromagnetic and gravitational fields. The sufficient mathematical conditions of levitation are obtained. It means that the regime of levitation could be realized from the position of the non-local hydrodynamics.

Keywords: *foundations of the theory of transport processes, generalized hydrodynamic equations, foundations of non-local physics, levitation.*

О НЕЛОКАЛЬНОЙ ТЕОРИИ ЛЕВИТАЦИИ

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В статье рассматриваются эффекты левитации с позиции обобщенной больцмановской физической кинетики, являющейся составной частью нелокальной физики. Теория позволяет идентифицировать условия, при которых возможно существование левитации в результате действия коррелированных электромагнитного и гравитационного полей. Сформулированы достаточные условия левитации. Установлена возможность описания левитации в рамках нелокальной гидродинамики.

Ключевые слова: *основания теории процессов переноса, обобщенные гидродинамические уравнения, основы нелокальной физики, левитация.*

1. Introduction

The phenomenon of levitation has attracted attention from philosophers and scientists in the past and now. How can levitation be possible? What power or agent accomplishes it? The most obvious explanation-the possession of a word of mystical power-is not interesting here for us. In spite of the tremendous recent advances, notably in power electronics, magnetic materials, on the application of electromagnetic suspension and levitation techniques to advanced ground transportation, physics of levitations needs in following significant investigations.

In this paper we revisit the levitation phenomenon using the generalized Bolzmann kinetics theory [1–5] which can represent the non-local physics of this levitation phenomenon.

The investigations of the levitation stability have a long history and are considered in details in [6–10]. As usual the problem review begins with the citation of the Earnshaw paper [6]. Earnshaw's theorem depends on a mathematical property of the 1/r type energy potential valid for magnetostatic and electrostatic events and gravitation. At any point where there is force balance is equal to zero, the equilibrium is unstable because there can be no local minimum in the potential energy. There must be some loopholes though, because magnets above superconductors and the magnet configuration do stably levitate including frogs [7] and toys like levitron (spinning magnet tops), flying globe and so on [11, 12]. It means that diamagnetic material can stabilize the levitation of permanent magnets. It is well known that the potential energy density of the magnetic field can be written as:

$$w_m = -\mathbf{M}\cdot\mathbf{B} \tag{1.1}$$

where **B** is magnetic induction, **M** is magnetization. Using the phenomenological relation

$$\mathbf{M} = \chi\mathbf{H}, \tag{1.2}$$

where χ is magnetic susceptibility, we have for the unit volume of a magnetic material

$$w_m = -\frac{\chi}{\mu\mu_0}B^2 \tag{1.3}$$

The force acting on the unit volume of a levitating object is

$$\mathbf{F} = \frac{\chi}{\mu_0\mu}\,\mathrm{grad}B^2, \tag{1.4}$$

if the phenomenological parameters are constant. Diamagnets (for which $\chi < 0$) are repelled by magnetic fields and attracted to field minima. As a result, diamagnets can satisfy the stability conditions [6 – 9] and the following conditions are exceptions to Earnshaw's theorem:

a) Diamagnetism which occurs in materials which have a relative permeability less than one. The result is that is eddy currents are induced in a diamagnetic material, it will repel magnetic flux.

b) The Meissner Effect which occurs in superconductors. Superconductors have zero internal resistance. As such induced currents tend to persist, and as a result the magnetic field they cause will persist as well.

c) As result of oscillations, when an alternating current is passed through an electromagnet, it behaves like a diamagnetic material.

d) Rotation: employed by the Levitron, it uses gyroscopic motion to overcome levitation instability.

e) Feedback can be used in conjunction with electromagnets to dynamically adjust magnetic flux in order to maintain levitation.

The main shortcoming of the Earnshaw theory consists in application of principles of local physics to the non-equilibrium non-local statistical systems.

The aim of this paper consists in application of the non-local physics methods to the effect of levitation. We intend to answer two questions:

1) Is it possible to formulate the sufficient conditions of levitation from the position of the unified non-local theory of transport processes (UNTT) [see, for example, 1–4].

2) Is it possible to speak about the mutual influence of electromagnetic field and gravitation in the frame of UNTT?

2. Basic equations

Non-local hydrodynamic equations have the form [1–4]:

(continuity equation for a mixture)

$$\frac{\partial}{\partial t}\left\{\rho-\sum_\alpha \tau_\alpha^{(0)}\left[\frac{\partial \rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0)\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho\mathbf{v}_0-\sum_\alpha \tau_\alpha^{(0)}\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0\mathbf{v}_0)+\overleftrightarrow{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial \mathbf{r}}-\right.\right.$$
$$\left.\left.-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}=0, \tag{2.1}$$

(motion equation)

$$\frac{\partial}{\partial t}\left\{\rho\mathbf{v}_0-\sum_\alpha \tau_\alpha^{(0)}\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}-\sum_\alpha \mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha^{(0)}\left(\frac{\partial \rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0)\right)\right]-$$
$$-\sum_\alpha \frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha^{(0)}\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}+$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho\mathbf{v}_0\mathbf{v}_0+p\overleftrightarrow{\mathrm{I}}-\sum_\alpha \tau_\alpha^{(0)}\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0\mathbf{v}_0+p_\alpha\overleftrightarrow{\mathrm{I}})+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0+2\overleftrightarrow{\mathrm{I}}\left(\frac{\partial}{\partial \mathbf{r}}\cdot(p_\alpha\mathbf{v}_0)\right)+\frac{\partial}{\partial \mathbf{r}}\cdot(\overleftrightarrow{\mathrm{I}}p_\alpha\mathbf{v}_0)-\right.\right.$$
$$\left.\left.-\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0-\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0[\mathbf{v}_0\times\mathbf{B}]\right]\right\}=0 \tag{2.2}$$

(energy equation)

$$\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p+\sum_\alpha \varepsilon_\alpha n_\alpha-\sum_\alpha \tau_\alpha^{(0)}\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\}+$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0+\mathbf{v}_0\sum_\alpha \varepsilon_\alpha n_\alpha-\sum_\alpha \tau_\alpha^{(0)}\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)\right.\right.$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\ddot{\mathrm{I}}+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\ddot{\mathrm{I}}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\ddot{\mathrm{I}}\right)-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}}- \qquad (2.3)$$
$$\left.\left.-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}-$$
$$-\mathbf{v}_0\cdot\sum_\alpha \rho_\alpha\mathbf{F}_\alpha^{(1)}+\sum_\alpha \tau_\alpha^{(0)}\mathbf{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha\ddot{\mathrm{I}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right]=0.$$

where $\mathbf{v}_0$ is the hydrodynamic velocity in the coordinate system at rest, ρ_α is the density of α-species, p is the pressure, $\ddot{\mathrm{I}}$ – unit tensore, $\mathbf{F}_\alpha^{(1)}$ is the force of the non-nonmagnetic origin acting on the unit of volume, ε_α is the internal energy of a particle of the α-species, τ is non-local parameter.

Important remarks:

1. Equations (2.1) – (2.3) should be considered as local approximation of non-local equations (NLE) written in the hydrodynamic form. NLE include quantum hydrodynamics of Schrödinger – Madelung as a deep particular case [4] and can be applied in the frame of the unified theory from the atom scale to the Universe evolution.

2. The basic system contains the cross terms for the forces of the mass and electro- magneto-dynamic origin. It means that the fluctuation of the gravitational field leads to the electro- magneto dynamical fluctuations and verse versa.

3. The upper index on the non-local parameter $\tau_\alpha^{(0)}$ underlines that non-local parameter is calculated in the local approximation of the non-local theory.

Sufficient conditions of levitation can be obtained from Eqs. (2.1) – (2.3) after equalizing all terms containing forces to zero. Namely, from the continuity equation

$$\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\sum_\alpha \tau_\alpha^{(0)}\left[\rho_\alpha\mathbf{F}_\alpha^{(1)}+\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]=0, \qquad (2.4)$$

from the motion equation follows:

$$\frac{\partial}{\partial t}\left\{\sum_\alpha \tau_\alpha^{(0)}\left[\rho_\alpha\mathbf{F}_\alpha^{(1)}+\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}-\sum_\alpha \mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha^{(0)}\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0)\right)\right]-\sum_\alpha\frac{q_\alpha}{m_\alpha}\left\{\tau_\alpha^{(0)}\left[\rho_\alpha\mathbf{F}_\alpha^{(1)}+\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\times$$
$$\times\mathbf{B}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\sum_\alpha \tau_\alpha^{(0)}\left[\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0+\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}+\frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\mathbf{v}_0+\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0[\mathbf{v}_0\times\mathbf{B}]\right]\right\}=0, \qquad (2.5)$$

and from the energy equation we find

$$\frac{\partial}{\partial t}\sum_\alpha \tau_\alpha^{(0)}\left(\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\sum_\alpha \tau_\alpha^{(0)}\left[\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0+p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}}+\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}+\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha+\right.\right.$$
$$\left.\left.+\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]+\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]+\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]+\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}- \qquad (2.6)$$
$$-\mathbf{v}_0\cdot\sum_\alpha \rho_\alpha\mathbf{F}_\alpha^{(1)}+\sum_\alpha \tau_\alpha^{(0)}\mathbf{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha\ddot{\mathrm{I}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right]=0.$$

From Eq. (2.4) we have

$$\sum_\alpha \tau_\alpha^{(0)}\left[\rho_\alpha\mathbf{F}_\alpha^{(1)}+\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]=\mathbf{L}(t), \qquad (2.7)$$

or

$$\sum_\alpha \tau_\alpha^{(0)}\left[\rho_\alpha\mathbf{F}_\alpha^{(1)}+\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]=\mathbf{L}\qquad, \qquad (2.8)$$

where $\mathbf{L}$ is constant vector. Let us introduce vector $\mathbf{L}_\alpha(t)$

$$\mathbf{L}_\alpha(t) = \tau_\alpha^{(0)}\left[\rho_\alpha \mathbf{F}_\alpha^{(1)} + \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right], \tag{2.9}$$

and rewrite now Eq. (2.5), which contains the density fluctuation [1]

$$\rho_\alpha^{fl} = \tau_\alpha^{(0)}\left(\frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha \mathbf{v}_0)\right). \tag{2.10}$$

We have

$$\begin{gathered}\frac{\partial}{\partial t}\mathbf{L}(t) - \sum_\alpha \mathbf{F}_\alpha^{(1)}\rho_\alpha^a - \sum_\alpha \frac{q_\alpha}{m_\alpha}\mathbf{L}_\alpha(t)\times\mathbf{B} + \mathbf{v}_0\left\{\frac{\partial}{\partial \mathbf{r}}\cdot\sum_\alpha \tau_\alpha^{(0)}\left[\mathbf{F}_\alpha^{(1)}\rho_\alpha + \frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\right]\right\} + \\ + \left(\sum_\alpha \tau_\alpha^{(0)}\left[\mathbf{F}_\alpha^{(1)}\rho_\alpha + \frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\right]\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0 + \sum_\alpha \tau_\alpha^{(0)}\left[\rho_\alpha \mathbf{F}_\alpha^{(1)} + \frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\right]\frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{v}_0 + \\ + \left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\left\{\sum_\alpha \tau_\alpha^{(0)}\left[\rho_\alpha \mathbf{F}_\alpha^{(1)} + \frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\right]\right\} = 0,\end{gathered} \tag{2.11}$$

where

$$\rho_\alpha^a = \rho_\alpha - \rho_\alpha^{fl} \;. \tag{2.12}$$

Using also (2.9), we find

$$\sum_\alpha \mathbf{F}_\alpha^{(1)}\rho_\alpha^a = \frac{\partial}{\partial t}\mathbf{L}(t) - \sum_\alpha \frac{q_\alpha}{m_\alpha}\mathbf{L}_\alpha(t)\times\mathbf{B} + \left(\mathbf{L}(t)\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0 + \mathbf{L}(t)\left(\frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{v}_0\right) + \left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{L}(t). \tag{2.13}$$

The vector product in Eq. (2.13) can be transformed as

$$\mathbf{L}_\alpha(t)\times\mathbf{B} = \tau_\alpha^{(0)}\rho_\alpha \mathbf{F}_\alpha^{(1)}\times\mathbf{B} - \tau_\alpha^{(0)}q_\alpha n_\alpha\left[\mathbf{v}_0 B^2 - \mathbf{B}(\mathbf{v}_0\cdot\mathbf{B})\right], \tag{2.14}$$

where $q_\alpha n_\alpha$ is the charge of α-species in the unit volume.

Taking into account the relations (2.10), (2.11), (2.12), we can realize the analogical transformation of the energy condition (2.6):

$$\begin{gathered}\mathbf{v}_0\cdot\sum_\alpha \mathbf{F}_\alpha^{(1)}\rho_\alpha^a = \frac{\partial}{\partial t}\sum_\alpha \tau_\alpha^{(0)}\left(\rho_\alpha \mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\right) + \sum_\alpha \tau_\alpha^{(0)}\mathbf{F}_\alpha^{(1)}\cdot\left[\rho_\alpha\frac{\partial \mathbf{v}_0}{\partial t} + \rho_\alpha\left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha \ddot{\mathrm{I}}\right] \\ + \frac{\partial}{\partial \mathbf{r}}\cdot\left\{\sum_\alpha \tau_\alpha^{(0)}\left[\begin{array}{l}\rho_\alpha \mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0 + p_\alpha \mathbf{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}} + \frac{1}{2}\rho_\alpha v_0^2 \mathbf{F}_\alpha^{(1)} + \frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha + \frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}] + \\ + \frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]\end{array}\right]\right\} + \\ + \frac{\partial}{\partial \mathbf{r}}\cdot\sum_\alpha \varepsilon_\alpha n_\alpha \mathbf{L}_\alpha - \sum_\alpha \tau_\alpha^{(0)}\mathbf{F}_\alpha^{(1)}\cdot\mathbf{L}_\alpha\end{gathered} \tag{2.15}$$

Equations (2.7), (2.13) and (2.15) define the system of the sufficient conditions for levitation.

The choice of the non-local parameter needs in the special consideration [3, 4]. The system of equations (2.1) – (2.3) convert in the system of quantum hydrodynamic equations by the suitable choice of the non-local parameter τ. The relation between τ and kinetic energy [3, 4] is used in quantum hydrodynamics

$$\tau = H/mu^2, \tag{2.16}$$

where u is the particle velocity, H is the coefficient of proportionality which reflects the state of the physical system. In the simplest case H is equal to the Plank constant $\hbar$ and the corresponding relation (2.16) correlates with the Heisenberg inequality. From the first glance the approximation (2.16) is distinguished radically from the kinetic relation known from the theory of the rarefied gases

$$\tau = \Pi\, \upsilon\rho/p, \tag{2.17}$$

which is used for the calculation of the non-local parameter in the macroscopic hydrodynamic case (υ is the kinematic viscosity). But it is not a case. In quantum approximation the value $\upsilon^{qu} = \hbar / m$ has the dimension $[cm^2/s]$ and can be called as quantum viscosity, for the electron species $\upsilon^{qu} = \hbar / m_e = 1.1577\ cm^2/s$. If we take into account that the value $p/\rho \sim \overline{V^2}$, then the interrelation of (2.16) and (2.17) becomes obvious.

3. Some particular cases of the levitation conditions

Write down the system of the sufficient levitation conditions for the quasi-stationary case neglecting dissipation and the space derivatives in Eq. (2.13). We find

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = -\sum_\alpha \frac{q_\alpha}{m_\alpha} \mathbf{L}_\alpha(t) \times \mathbf{B}. \tag{3.1}$$

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = \mathbf{B} \times \sum_\alpha \tau_\alpha^{(0)} \frac{q_\alpha}{m_\alpha}\left[\rho_\alpha \mathbf{F}_\alpha^{(1)} + \frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0 \times \mathbf{B}\right] \tag{3.2}$$

Introducing the current density

$$\mathbf{j}_\alpha = \tau_\alpha^{(0)} q_\alpha n_\alpha \mathbf{v}_0 \quad , \tag{3.3}$$

one obtains

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = \mathbf{B} \times \sum_\alpha \tau_\alpha^{(0)} q_\alpha n_\alpha \mathbf{F}_\alpha^{(1)} + \mathbf{B} \times \sum_\alpha \frac{q_\alpha}{m_\alpha}[\mathbf{j}_\alpha \times \mathbf{B}] \tag{3.4}$$

The right-hand-side of Eq. (3.4) contains the cross terms for the forces of the mass and electro- magneto-dynamic origin. The last term in Eq. (3.4) can be written also in the form

$$\mathbf{B} \times \sum_\alpha \frac{q_\alpha}{m_\alpha}[\mathbf{j}_\alpha \times \mathbf{B}] = \sum_\alpha \rho_\alpha \tau_\alpha^{(0)} \left[\frac{q_\alpha}{m_\alpha}\right]^2 \left\{\mathbf{v}_0 B^2 - \mathbf{B}(\mathbf{v}_0 \cdot \mathbf{B})\right\}. \tag{3.5}$$

Taking into account (2.16), (2.17) it is naturally to suppose that

$$k_B T \tau_\alpha^{(0)} \ge \hbar , \tag{3.6}$$

introduce now

$$\tau_\alpha^{(0)} = A \frac{\hbar}{k_B T}, \tag{3.7}$$

where A is a parameter which leads to appearance the effective temperature T_{eff}. Other approximations can be used, for example

$$\tau_\alpha^{(0)} = \frac{\hbar}{k_B T_{\alpha,eff}} . \tag{3.8}$$

Let us consider now other particular case when $\mathbf{v}_0 = 0$. In equations (2.7), (2.13) and (2.15) we conserve the terms up to the $\tau_\alpha^{(0)}$ order. From Eq. (2.13) follows

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = \frac{\partial}{\partial t}\mathbf{L}(t) - \sum_\alpha \frac{q_\alpha}{m_\alpha}\mathbf{L}_\alpha(t) \times \mathbf{B}, \tag{3.9}$$

where now

$$\mathbf{L}_\alpha(t) = \tau_\alpha^{(0)} \rho_\alpha \mathbf{F}_\alpha^{(1)}, \tag{3.10}$$

$$\mathbf{L}(t) = \sum_\alpha \tau_\alpha^{(0)} \rho_\alpha \mathbf{F}_\alpha^{(1)} . \tag{3.11}$$

Then

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = \frac{\partial}{\partial t}\sum_\alpha \tau_\alpha^{(0)} \rho_\alpha^a \mathbf{F}_\alpha^{(1)} - \sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} \mathbf{F}_\alpha^{(1)} \times \mathbf{B} . \tag{3.12}$$

Introduce the explicit expression for the mass force

$$\mathbf{F}_\alpha^{(1)} = \mathbf{g} + \frac{q_\alpha}{m_\alpha}\mathbf{E} \tag{3.13}$$

in Eq. (3.9)

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = \frac{\partial}{\partial t}\sum_\alpha \tau_\alpha^{(0)} \rho_\alpha^a \left(\mathbf{g} + \frac{q_\alpha}{m_\alpha}\mathbf{E}\right) - \sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} \left(\mathbf{g} + \frac{q_\alpha}{m_\alpha}\mathbf{E}\right) \times \mathbf{B} \tag{3.14}$$

where $n_\alpha^a = \rho_\alpha^a / m_\alpha$. From Eq. (3.14) follows

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = \frac{\partial}{\partial t}\sum_\alpha \tau_\alpha^{(0)} \rho_\alpha^a \mathbf{g} + \frac{\partial}{\partial t}\left[\mathbf{E}\sum_\alpha \tau_\alpha^{(0)} n_\alpha^a q_\alpha\right] - \mathbf{g} \times \mathbf{B}\sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} - \mathbf{E} \times \mathbf{B}\sum_\alpha \left(\frac{q_\alpha}{m_\alpha}\right)^2 \rho_\alpha^a \tau_\alpha^{(0)} . \tag{3.15}$$

Let us introduce in Eq. (3.15) the Umov – Pointing vector $\mathbf{S}$ and Alexeev vector $\mathbf{S}_A$ in the forms

$$\mathbf{S} = \mathbf{E} \times \mathbf{B}, \tag{3.16}$$

$$\mathbf{S}_A = \mathbf{g} \times \mathbf{B} . \tag{3.17}$$

In this case

$$\sum_\alpha \mathbf{F}_\alpha^{(1)} \rho_\alpha^a = \frac{\partial}{\partial t}\left(\sum_\alpha \tau_\alpha^{(0)} \rho_\alpha^a \mathbf{g}\right) + \frac{\partial}{\partial t}\left[\mathbf{E}\sum_\alpha \tau_\alpha^{(0)} n_\alpha^a q_\alpha\right] - \mathbf{S}_A \sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} - \mathbf{S}\sum_\alpha \left(\frac{q_\alpha}{m_\alpha}\right)^2 \rho_\alpha^a \tau_\alpha^{(0)} . \tag{3.18}$$

For the approximation (3.7) one obtains

$$\sum_\alpha \mathbf{F}_\alpha^{(1)}\rho_\alpha^a = A\frac{\hbar}{k_B}\left[\frac{\partial}{\partial t}\left(\frac{\rho^a}{T}\mathbf{g}\right)+\frac{\partial}{\partial t}\left(\mathbf{E}\frac{Q^a}{T}\right)-\mathbf{S}_A\frac{Q^a}{T}-\mathbf{S}\frac{1}{T}\sum_\alpha\left(\frac{q_\alpha}{m_\alpha}\right)^2\rho_\alpha^a\right], \tag{3.19}$$

where the average charge density is introduced

$$Q^a = \sum_\alpha q_\alpha n_\alpha^a . \tag{3.20}$$

The analogical transformations of the energy condition (2.15) can be realized for this particular case when $\mathbf{v}_0^a = 0$. Namely

$$-\sum_\alpha \tau_\alpha^{(0)}\mathbf{F}_\alpha^{(1)}\cdot\sum_\alpha \mathbf{F}_\alpha^{(1)}\rho_\alpha^a = \sum_\alpha \tau_\alpha^{(0)}\mathbf{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha^a \ddot{\mathrm{I}}\right]+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\sum_\alpha \tau_\alpha^{(0)}\left[p_\alpha^a\mathbf{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}}+\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha^a\right]\right\}+$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\sum_\alpha \varepsilon_\alpha n_\alpha^a \tau_\alpha^{(0)}\mathbf{F}_\alpha^{(1)}. \tag{3.21}$$

For the approximation (3.7) we find from Eq. (3.21)

$$\frac{\partial}{\partial \mathbf{r}}\cdot\sum_\alpha\left[\frac{5}{2}p_\alpha^a+\varepsilon_\alpha n_\alpha^a\right]\mathbf{F}_\alpha^{(1)}-\sum_\alpha\left(\frac{5}{2}p_\alpha^a+\varepsilon_\alpha n_\alpha^a\right)\mathbf{F}_\alpha^{(1)}\cdot\frac{\partial \ln T}{\partial \mathbf{r}}=$$
$$=-\mathbf{F}^{(1)}\cdot\sum_\alpha \rho_\alpha^a\mathbf{F}_\alpha^{(1)}-\sum_\alpha \mathbf{F}_\alpha^{(1)}\cdot\frac{\partial p_\alpha^a}{\partial \mathbf{r}}, \tag{3.22}$$

where $\mathbf{F}^{(1)} = \sum_\alpha \mathbf{F}_\alpha^{(1)} = \sum_\alpha\left(\mathbf{g}+\frac{q_\alpha}{m_\alpha}\mathbf{E}\right)$. Eq. (3.22) should be considered as a relation defining the energy consumption needed for the levitation.

From (3.13) follows a relation

$$\sum_\alpha \mathbf{F}_\alpha^{(1)}\rho_\alpha^a = \mathbf{g}\rho^a+\mathbf{E}\sum\frac{q_\alpha}{m_\alpha}\rho_\alpha^a = \mathbf{g}\rho^a+\mathbf{E}\sum_\alpha q_\alpha n_\alpha^a = \mathbf{g}\rho^a+\mathbf{E}Q^a, \tag{3.23}$$

which can be used for the transformation of Eq. (3.22). For a tentative estimate we can omit the derivatives of the logarithmic terms and the time derivatives for a quasi-neutral media. As a result from (3.22)

$$\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\mathbf{g}\sum_\alpha\left[\frac{5}{2}p_\alpha^a+\varepsilon_\alpha n_\alpha^a\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\mathbf{E}\sum_\alpha\frac{q_\alpha}{m_\alpha}\left[\frac{5}{2}p_\alpha^a+\varepsilon_\alpha n_\alpha^a\right]\right\}=$$
$$=-\mathbf{F}^{(1)}\cdot\left[\rho^a\mathbf{g}+\mathbf{E}Q^a\right]-\mathbf{g}\cdot\frac{\partial p^a}{\partial \mathbf{r}}-\mathbf{E}\cdot\sum_\alpha\frac{q_\alpha}{m_\alpha}\frac{\partial p_\alpha^a}{\partial \mathbf{r}}. \tag{3.24}$$

For a quasi-neutral media

$$Q^a = 0, \tag{3.25}$$

then

$$\frac{\partial}{\partial \mathbf{r}}\cdot\left[\mathbf{g}\left(\frac{5}{2}p^a+\Xi\right)\right]+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\mathbf{E}\sum_\alpha\frac{q_\alpha}{m_\alpha}\left[\frac{5}{2}p_\alpha^a+\Xi_\alpha\right]\right\}=$$
$$=-\rho^a\mathbf{F}^{(1)}\cdot\mathbf{g}\;-\mathbf{E}\cdot\sum_\alpha\frac{q_\alpha}{m_\alpha}\frac{\partial p_\alpha^a}{\partial \mathbf{r}}-\mathbf{g}\cdot\frac{\partial p^a}{\partial \mathbf{r}}, \tag{3.26}$$

where

$$\Xi = \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha}^{a}, \quad \Xi_{\alpha} = \varepsilon_{\alpha} n_{\alpha}^{a} \cdot \tag{3.27}$$

Let us obtain a tentative estimate from (3.19) for the quasi-stationary case in a quasi-neutral media. From (3.18) for the case under consideration we have

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^{a} = -\mathbf{S}_{A} \sum_{\alpha} q_{\alpha} n_{\alpha}^{a} \tau_{\alpha}^{(0)} - \mathbf{S} \sum_{\alpha} \left(\frac{q_{\alpha}}{m_{\alpha}} \right)^{2} \rho_{\alpha}^{a} \tau_{\alpha}^{(0)}. \tag{3.28}$$

Only in the case when the non-local parameter does not depend on the kind of species the first term of the right-hand-side of Eq. (3.28) is equal to zero taking into account (3.25).

From (3.8), (3.23), (3.25), (3.28) we find

$$\rho^{a} \mathbf{g} = -\mathbf{S}_{A} \sum_{\alpha} q_{\alpha} n_{\alpha}^{a} \tau_{\alpha}^{(0)} - \mathbf{S} \sum_{\alpha} \left(\frac{q_{\alpha}}{m_{\alpha}} \right)^{2} \rho_{\alpha}^{a} \tau_{\alpha}^{(0)}, \tag{3.29}$$

or

$$\rho^{a} \mathbf{g} = -A \frac{\hbar}{k_{B}} \mathbf{S} \frac{1}{T} \sum_{\alpha} \left(\frac{q_{\alpha}}{m_{\alpha}} \right)^{2} \rho_{\alpha}^{a} \tag{3.30}$$

in the case of (3.7) approximation. Relation leads in SI to the estimate

$$\rho^{a} \mathbf{g} \cong -A \frac{\mathbf{S}}{T} \cdot 2.138 \cdot 10^{-19} n_{e}^{a} \tag{3.31}$$

or

$$\rho^{a} \mathbf{g} \cong 2.138 \cdot 10^{-19} \frac{A}{T} [\mathbf{B} \times \mathbf{E}] n_{e}^{a} \cdot \tag{3.32}$$

The following table lists known examples of number densities at 1 atm and 20 °C, unless otherwise noted.

Molecular number density and related parameters of some materials

Material	Number density (n)	Density (p)
Units	(10^{27} m^{-3}) or (10^{21} cm^{-3})	(10^3 kg/m^3) or (g/cm^3)
dry air	0.02504	1.2041×10^{-3}
water	33.3679	0.99820
diamond	176.2	3.513

Let us use now (1.4) and the obvious phenomenological condition of the force balance (see also [9]) we have

$$\mathbf{F} = \frac{\chi}{\mu_0 \mu} \mathrm{grad} B^2 = \rho g \hat{\mathbf{e}}_z, \tag{3.33}$$

where ρ is the mass density of the material to be levitated and $\hat{\mathbf{e}}_Z$ is the unit vector in the vertical direction, magnetic susceptibility χ is negative for diamagnetic materials. In the frame of the phenomenological description of the magnetic and gravitational field we have

$$w = w_m + w_g = -\frac{\chi}{\mu \mu_0} B^2 + \rho g z \tag{3.34}$$

A necessary condition for stability is

$$\int_S \mathbf{F} \cdot d\mathbf{s} < 0, \tag{3.35}$$

where S is any small closed surface surrounding the equilibrium point. It leads to the condition

$$\mathrm{div}\ \mathbf{F} < 0 \cdot \tag{3.36}$$

This relation leads to the stability condition

$$\Delta w = \mathrm{divgrad}\ w = \mathrm{divgrad}\ w_m = -\frac{\chi}{\mu \mu_0} \Delta B^2 = -\mathrm{div} \mathbf{F} > 0, \tag{3.37}$$

if $\chi < 0$ (diamagnetic materials) and $\Delta B^2 > 0$. The corresponding stability investigation from the phenomenological point of view was realized in [6].

From the relation (3.33) follows ($\mu \sim 1$)

$$\rho g = 2 \frac{\chi}{\mu_0} B \frac{\partial B}{\partial z}, \tag{3.38}$$

and from (3.29)

$$\rho^a \mathbf{g} = -\mathbf{S}\tau^{(0)} \sum_\alpha \left(\frac{q_\alpha}{m_\alpha} \right)^2 \rho_\alpha^a , \tag{3.39}$$

if the non-local parameter does not depend on the sort of species α. After equalizing the right-hand-sides of relations (3.38) and (3.39) one obtains

$$\tau^{(0)} \mu_0 E \frac{q_e}{2m_e} q_e n_e^a = \chi \frac{\partial B}{\partial z} , \tag{3.40}$$

because

$$\sum_\alpha \left(\frac{q_\alpha}{m_\alpha} \right)^2 \rho_\alpha^a \cong \left(\frac{q_i}{m_i} \right)^2 \rho^a + \left(\frac{q_e}{m_e} \right)^2 \rho_e^a = \frac{q_i^2}{m_i} n_i^a + \frac{q_e^2}{m_e} n_e^a \cong \left(\frac{q_e}{m_e} \right)^2 \rho_e^a . \tag{3.41}$$

Let us introduce the character length l_m

$$l_m = \mu_0 \frac{q_e^2}{2m_e} , \tag{3.42}$$

hence from (3.40), (3.42)

$$l_m E n_e^a = -|\chi| \frac{1}{\tau^{(0)}} \frac{\partial B}{\partial z} . \tag{3.43}$$

Introduce the electromotive force (EMF) for a particle

$$\mathrm{E}_{ind} = l_m E , \tag{3.44}$$

and for n_e^a particles

$$\mathrm{E}_{ind,n} = l_m E n_e^a . \tag{3.45}$$

Hence from (3.43), (3.45) we find

$$\mathrm{E}_{ind,n} = -\frac{|\chi|}{\tau^{(0)}} \frac{\partial B}{\partial z} . \tag{3.46}$$

Formally Eq. (3.46) can be written in the form of Faraday's law of induction, the most widespread version of this law states that the induced electromotive force in any closed circuit is equal to the rate of change of the magnetic flux through the circuit:

$$\mathrm{E}_{ind} = -\frac{\partial \Phi_B}{\partial t} , \tag{3.47}$$

where Φ_B is the magnetic flux. This version of Faraday's law strictly holds only when the closed circuit is a loop of infinitely thin wire and is invalid in some other circumstances. Nevertheless formally

$$\mathrm{E}_{ind,n} = -|\chi| \frac{1}{\tau^{(0)}} \frac{\partial B}{\partial t} \frac{\partial t}{\partial z} , \tag{3.48}$$

or

$$\mathrm{E}_{ind,n} = -|\chi| \frac{1}{\tau^{(0)}} \frac{\partial B}{\partial t} \frac{1}{v_m} . \tag{3.49}$$

After introduction of the character the counter square

$$S_m = \frac{|\chi|}{\tau^{(0)} v_m n_e^a} , \tag{3.50}$$

we reach the relation in the form of Faraday's law

$$\mathrm{E}_{ind} = -\frac{\partial \Phi_B}{\partial t} . \tag{3.51}$$

Let us consider now the force balance (3.18) for the quasi-stationary case by the absence of the external electric field. We have

$$\rho^a \mathbf{g} = -\mathbf{S}_A \sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} - \mathbf{S} \sum_\alpha \left(\frac{q_\alpha}{m_\alpha} \right)^2 \rho_\alpha^a \tau_\alpha^{(0)} , \tag{3.52}$$

where (see (3.16), (3.17)) $\mathbf{S} = \mathbf{E} \times \mathbf{B}$, $\mathbf{S}_A = \mathbf{g} \times \mathbf{B}$. The effect of polarization leads to diminishing of the external intensity of electric field. We suppose that the external intensity of electric field is equal to zero. It means that relation (3.52) should be written in the form

$$\rho^a \mathbf{g} = \mathbf{B} \times \mathbf{g} \sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} + \mathbf{B} \times \mathbf{E}' \sum_\alpha \left(\frac{q_\alpha}{m_\alpha} \right)^2 \rho_\alpha^a \tau_\alpha^{(0)} , \tag{3.53}$$

where the electric intensity **E**' reflects the polarization effect. We find

$$\rho^a \mathbf{g} = \mathbf{B} \times \mathbf{g} \sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} + \mathbf{B} \times \mathbf{E}' \sum_\alpha \left(\frac{q_\alpha}{m_\alpha} \right) q_\alpha n_\alpha^a \tau_\alpha^{(0)} , \tag{3.54}$$

or

$$\rho^a \mathbf{g} = \mathbf{B} \times \left\{ \sum_\alpha q_\alpha n_\alpha^a \tau_\alpha^{(0)} \left[\mathbf{g} + \mathbf{E}' \frac{q_\alpha}{m_\alpha} \right] \right\} . \tag{3.55}$$

Let us consider the two component mixture of negative and positive charged particles:

$$\rho^a \mathbf{g} = \mathbf{B} \times \mathbf{g} \left(q_i n_i^a \tau_i^{(0)} + q_e n_e^a \tau_e^{(0)} \right) + \mathbf{B} \times \mathbf{E}' \left(q_i^2 n_i^a \tau_i^{(0)} \frac{1}{m_i} + q_e^2 n_e^a \tau_e^{(0)} \frac{1}{m_e} \right) . \tag{3.56}$$

It is naturally to suppose that for the quasi-neutral matter

$$q_i n_i^a = |q_e| n_e^a \;,\; \tau_e^{(0)} >> \tau_i^{(0)} \;,\; m_e << m_a \;.$$

In this case we have from (3.56)

$$\left(m_e n_e^a + m_i n_i^a\right)\mathbf{g} = \mathbf{B}\times\mathbf{g}\left(q_i n_i^a \tau_i^{(0)} + q_e n_e^a \tau_e^{(0)}\right) + \mathbf{B}\times\mathbf{E}'\left(q_e^2 n_e^a \tau_e^{(0)} \frac{1}{m_e}\right). \qquad (3.57)$$

But $n_e^a \sim n_i^a$, then

$$m_i n_i^a \mathbf{g} = \mathbf{B}\times\mathbf{g} q_e n_e^a \tau_e^{(0)} + \mathbf{B}\times\mathbf{E}'\left(q_e^2 n_e^a \tau_e^{(0)} \frac{1}{m_e}\right),\; m_i \mathbf{g}\frac{1}{\tau_e^{(0)}} = \mathbf{B}\times\mathbf{g} q_e + \mathbf{B}\times\mathbf{E}'\frac{q_e^2}{m_e} \qquad (3.58)$$

Let us consider the limit cases of relation (3.58):

1. $\tau_e = 0$; levitation of the *arbitrary* mass can be realized in the frame of the local description, only if $\mathbf{g} = 0$.

2. $\tau_e \to \infty$. We have

$$\mathbf{B}\times\mathbf{g}|q_e| = -\mathbf{B}\times\mathbf{E}'\frac{q_e^2}{m_e}, \qquad (3.59)$$

which leads to the obvious relation valid in the considering limit case

$$\mathbf{g} = -\mathbf{E}'\frac{q_e^2}{|q_e| m_e} \qquad (3.60)$$

or

$$m_e \mathbf{g} = -|q_e|\mathbf{E}' \cdot \qquad (3.61)$$

As we see gravitation leads to the polarization of matter.

Relation (3.61) can be considered as a direct estimation of the electric intensity **E**' for this case

$$\mathbf{E}' = -\frac{m_e}{|q_e|}\mathbf{g}.$$

The value $\frac{|q_e|}{m_e} = q_{ratio}$ is known as an electron-charge mass ratio. Then

$$\mathbf{E}' = -\frac{1}{q_{ratio}}\mathbf{g}. \qquad (3.62)$$

The simple estimation is valid

$$E' = \frac{g}{q_{ratio}} = 0.557\cdot 10^{-10}\frac{V}{m}. \qquad (3.63)$$

Taking into account the last result we have a new view of the oil drop experiment performed by Robert A. Millikan and Harvey Fletcher in 1909 to measure the charge of the electron. The experiment entailed balancing the downward gravitational force F_r with the upward drag acting on the spherical droplet (ρ, ρ_0 are the densities of oil and air correspondingly)

$$F_r = \frac{4}{3}\pi r^3(\rho - \rho_0)g \qquad (3.64)$$

and electric forces on tiny charged droplets of oil suspended between two metal electrodes;

$$F_r = Q_r E \,. \qquad (3.65)$$

The density of the oil was known. Therefore the droplets' masses, their gravitational and buoyant forces could be determined from observed radii. Using a known electric field, Millikan and Fletcher could determine the charge on oil droplets in mechanical equilibrium. By repeating the experiment for many droplets, they confirmed that the charges were all multiples of some fundamental value q_e . They proposed that this was the charge of a single electron.

Obviously Eq. (3.65) corresponds to Eq. (3.61). Then from position of non-local physics Mellikan experiment reflects the polarized matter levitation realized for the deep particular case when non-local parameter $\tau_e \to \infty$.

4. Conclusion

The following conclusions of the principal significance can be done:

1. The levitation effects are the direct consequence of the non-local equations (2.1) – (2.3).
2. The sufficient conditions of levitation are the particular case of Eqs. (2.1) – (2.3).
3. The strict theory of levitation can be constructed only in the frame of non-local physics.
4. Fluctuations of the gravitational field lead to the electro-magneto dynamical fluctuations and verse versa. This fact can effect on the work of electronic devices during the evolution of the wave atmospheric fronts.
5. Levitation effects are connected not only with the electro-magnetic energy flux **S**, but also with the cross flux $\mathbf{S}_A$.

6. Usual local conditions of levitation are the deep particular cases of the non-local theory.

7. Mellikan experiment corresponds to the deep particular case when non-local parameter $\tau_e \to \infty$.

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