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MATHEMATICS METHODS AND INFORMATION SYSTEMS IN CHEMICAL TECHNOLOGY

THERMAL SHOCK AND DYNAMIC THERMOELASTICITY

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This paper considers the problem of thermal shock to a massive body in different conditions of heating and cooling; the most dangerous mode of heating was identified, the influence of inertial effects on the value of emerging thermal stress was investigated.
A new equation of compatibility of stress with the inertial effects, which generalizes the known Beltrami-Mitchell relation for quasi-static cases, was obtained by methods of the tensor algebra. The theory of heat stroke solids was developed in terms of dynamic problems of thermoelasticity in different forms of heat stress: temperature heating; thermal heating; heating medium. The equations to calculate the jumps on the front of thermoelastic waves were obtained. The most dangerous mode of heat stroke was identified.
The effect of relaxation in thermal problems was described in the context of the investigation of thermal stress state of a massive body. It was shown, that an increase in relaxation time, i.e. heating rates of the boundary surface of the body, causes a reduction of thermal stress maxima. The original results of the thermal reaction of a solid to cooling were obtained; it was shown that, in comparison with the heating mode, the cooling mode is more devastating, especially for near-surface layers of solids. The role of the relaxation temperature in the cooling mode was identified. New functional structures were proposed as analytical solutions to the major dynamic problems of thermomechanic on the basis of the use of the Kar functions, which are relatively new.

Keywords: *heat stroke, heat mode of loading, an abrupt change in temperature, voltage.*

Introduction

Thermal shock is one of central problems in thermomechanics in the context of the creation of powerful energy radiators and their use in various technological operations. Research along this line based on models of dynamic and quasistatic thermoelasticity reached an advanced stage of development: physical regularities of the thermally stressed state in isotropic and anisotropic elastic bodies based on classical Fourier and Maxwell-Cattaneo-Lykov phenomenologies concerning the finite speed of heat propagation in solid bodies were studied; a generalized theory of thermomechanical fields interference with fields of various physical nature (electric, magnetic) was developed; defining relations of the linearized theory taking into account thermal memory were formulated; interrelation of macroscopic behavior of a continuous medium with its internal state parameters and the rate of their change in time was established. Systematization of the results accumulated in this area of thermomechanics is presented in reviews [1, 2] and book [3].

Defining relations of dynamic thermoelasticity. Let us assume that D is a finite or partially limited convex region of space $M(x,y,z)$ existing in a thermally stressed state; S is a piecewise smooth surface, which confines region D; $\vec{n}$ is an external normal to S – a vector continuous in points of S; $T(M,t)$ is distribution of temperature in region D at $t>0$; T_0 is initial temperature, at which region D exists in a non-prestressed and non-deformable state.

Let us assume that $\sigma_{ij}(M,t)$, $\varepsilon_{ij}(M,t)$, $U_i(M,t)$, $(i,j=x,y,z)$ are, respectively, components of the stress tensors, of the deformation tensors and of the displacement vector. These components fit the main equations of (uncoupled) thermoelasticity [3]: motion equations (taking into account volume forces $F_i(M,t)$, geometrical relationships and the physical equations (in index designations):

$$\sigma_{ij,j}(M,t)+F_i(M,t)=\rho\ddot{U}_i(M,t); \tag{1}$$

$$\varepsilon_{ij}(M,t)=(1/2)\left[U_{i,j}(M,t)+U_{j,i}(M,t)\right]; \tag{2}$$

$$\sigma_{ij}(M,t)=2\mu\varepsilon_{ij}(M,t)+\left[\lambda\varepsilon_{ii}(M,t)-(3\lambda+2\mu)\alpha_T\left(T(M,t)-T_0\right)\right]\delta_{ij}; \tag{3}$$

$M \in D$, $t > 0$,

where ρ is density; $\mu = G$, $\lambda = 2G\nu/(1-2\nu)$ is Lamé's isothermal coefficient; ν is Poisson's coefficient, while $2G(1+\nu) = E$; E is Young's modulus; G is shearing modulus; α_T is coefficient of thermal linear expansion; δ_{ij} is Kronecker symbol; $\tilde{e}(M,t) = U_{i,j}(M,t) = \varepsilon_{ii}(M,t)$ is volume deformation connected with the sum of normal stresses according to the following formula:

$$\tilde{e}(M,t) = \frac{1-2\nu}{E}\tilde{\sigma}(M,t) + 3\alpha_T\left[T(M,t) - T_0\right]. \quad (4)$$

Excluding the components of the displacement vector in (2) we obtain the known deformations compatibility equation in the form $\gamma_{pmj}\gamma_{qni}\varepsilon_{ij,mn}(M,t) = 0$, where γ_{ijk} is alternative (antisymmetric) tensor $(p,q,m,n = x,y,z)$. This equation can be written in more detail:

$$\varepsilon_{ij,mn} - \varepsilon_{im,jn} - \varepsilon_{nj,mi} - \varepsilon_{nm,ji} = 0. \quad (5)$$

Let us express deformations in terms of stresses from (3):

$$\varepsilon_{ij}(M,t) = \frac{1+\nu}{E}\sigma_{ij}(M,t) - \frac{\nu}{E}\sigma_{nn}(M,t)\delta_{ij} + \alpha_T\left[T(M,t) - T_0\right]\delta_{ij}. \quad (6)$$

Let us contract the tensors in (5) by indexes: $\varepsilon_{ij,nn} - \varepsilon_{in,jn} - \varepsilon_{nj,ni} - \varepsilon_{nn,ji} = 0$ and substitute the right parts of equation (6). Transformations with the use of (1), (2) and tensor algebra properties give the following basic equation of dynamic thermoelasticity with stresses:

$$\begin{aligned}&(1+\nu)\sigma_{ij,nn}(M,t) + \sigma_{nn,ji}(M,t) + \frac{\nu(1+\nu)}{(1-\nu)}F_{n,n}(M,t)\delta_{ij} + (1+\nu)\left[F_{i,j}(M,t) + F_{j,i}(M,t)\right] + \\ &+ E\alpha_T\left[\frac{1+\nu}{1-\nu}\left(T(M,t) - T_0\right)_{,nn}\delta_{ij} + \left(T(M,t) - T_0\right)_{,ij}\right] = \\ &= \frac{(1+\nu)}{2G}\frac{\partial^2}{\partial t^2}\left[2\sigma_{ij}(M,t) - \frac{\nu}{1-\nu}\sigma_{nn}(M,t)\delta_{ij} + \frac{2G(2+\nu)}{1-\nu}\alpha_T\left(T(M,t) - T_0\right)\delta_{ij}\right], \ \mathrm{M} \in \mathrm{D}, \mathrm{t} > 0.\end{aligned} \quad (7)$$

Equation (7) is the generalized Beltrami-Michell equation for dynamic tasks. This case was considered for the first time by V. Novatsky with the use of elastokinetics equations with stresses. However, the end result has a form differing from (7) and less convenient for practical applications. In this context equation (7) itself is of interest for thermomechanics.

The thermally stressed state of region D can arise in various modes of heat impact on S boundary creating thermal shock. Among these modes there are cases most commonly occurring in practice: temperature heating $T(M,t) = T_C$, $M \in S$, $t > 0$ $(T_C > T_o)$; thermal heating $\frac{\partial T(M,t)}{\partial n} = -\frac{1}{\lambda_T}q_0$, $M \in S$, $t > 0$, (λ_T is the thermal conductivity of the material; q_0 is the size of the thermal flow); heating by the medium $\frac{\partial T(M,t)}{\partial n} = -h\left[T(M,t) - T_C\right]$, $M \in S$, $t > 0$ (h is the relative coefficient of heat exchange; T_C is ambient temperature $(T_C > T_o)$). Also cases of uniform cooling can be equally considered.

In the context of using equation (7) let us consider a case which is important for many practical applications. Elastic half-space $z \geq l$ originally existing at temperature $T_0 \geq 0$ is exposed on the boundary to various modes of thermal influence creating thermal shock, namely: 1) temperature heating $T(z,t)\big|_{z=l} = T_C$, $t > 0$ $(T_C > T_o)$; 2) thermal heating $\frac{\partial T(z,t)}{\partial z}\big|_{z=l} = -\frac{1}{\lambda_T}q_0$ $t > 0$; 3) heating by the medium $\frac{\partial T(z,t)}{\partial z}\big|_{z=l} = h\left[T(z,t)\big|_{z=l} - T_C\right]$, $t > 0$. In these conditions at one-dimensional motion $U_x = U_y = 0$; $U_z = U_z(z,t)$; $\varepsilon_{zx} = \varepsilon_{zy} = \varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{xy} = 0$; $\varepsilon_{zz} = \varepsilon_{zz}(z,t)$; stresses $\sigma_{ij} = 0$ in case of $i \neq j$ and $\sigma_{ij} = \sigma_{ij}(z,t)$ in case of $i = j$. If volume forces are absent and the boundary of the body is free from stress, equation (7) leads to the following dynamic problem of thermoelasticity:

$$\frac{\partial^2\sigma_{zz}}{\partial z^2} - \frac{1}{\upsilon_p^2}\frac{\partial^2\sigma_{zz}}{\partial t^2} = \frac{(1+\nu)}{(1-\nu)}\alpha_T\rho\frac{\partial^2 T(z,t)}{\partial t^2}, \ z > l, \ t > 0; \quad (8)$$

$$\sigma_{zz}\big|_{t=o} = 0, \ \frac{\partial\sigma_{zz}}{\partial t}\big|_{t=0} = 0, \ z \geq l; \quad (9)$$

$$\sigma_{zz}(z,t)\big|_{z=l} = \sigma_{zz}(z,t)\big|_{z=\infty}, \ t > 0; \quad (10)$$

where

$$\upsilon_p = \sqrt{\frac{2G(1-\nu)}{\rho(1-2\nu)}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}. \quad (11)$$

– the speed of the expansion wave propagation in the elastic medium, close to the speed of sound. According to (3)–(4) other non-zero components of the stress tensor are given by:

$$\sigma_{xx}(z,t)=\sigma_{yy}(z,t)=\frac{\nu}{1-\nu}\sigma_{zz}(z,t)-\frac{E\alpha_T\left[T(z,t)-T_0\right]}{1-\nu}, \qquad (12)$$

and

$$\varepsilon_{zz}(z,t)=\frac{1-2\nu}{2G(1-\nu)}\sigma_{zz}(z,t)+\frac{(1+\nu)}{(1-\nu)}\alpha_T\left[T(z,t)-T_0\right]\cdot \qquad (13)$$

Equation (8) was obtained for the first time by Danilovskaya directly from relationships (1)–(3) and, irrespectively of her, by T. Mura, who apparently did not know about the previous and more general work of Danilovskaya.

Let us first consider heating.

Temperature function *T(z, t)* in (8)–(13) is the task solution:

$$\left.\begin{array}{l}\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial z^2}, z>l, t>0\\ T(z,t)=T_0, z\ge l, |T(z,t)|<\infty, z\ge l, t\ge 0\end{array}\right\} \qquad (14)$$

including also one of the three types of the above boundary conditions. In order to carry out a numerical experiment let us enter dimensionless variables:

$$\xi=\frac{\upsilon_p(z-l)}{a}\ ,\ \tau=\frac{\upsilon_p^2 t}{a},\ Bi^*=\frac{ha}{\upsilon_p};$$

$$W(\xi,\tau)=\begin{cases}\dfrac{T(z,t)-T_0}{T_C-T_0}, & \text{1st and 3rd boundary value problems;}\\ \dfrac{T(z,t)-T_0}{(q_0/\lambda_T)(a/\upsilon_p)}, & \text{2nd boundary value problem;}\end{cases}$$

$$\sigma_{\xi\xi}(\xi,\tau)=\begin{cases}\dfrac{\sigma_{zz}(z,t)}{\tilde{s}(T_C-T_0)}, & \text{1st and 3rd boundary value problems;}\\ \dfrac{\sigma_{zz}(z,t)}{\tilde{s}(q_0/\lambda_T)(a/\upsilon_p)}, & \text{2nd boundary value problem;}\end{cases}$$

where $\tilde{s}=\alpha_T E/(1-2\nu)$. Let us introduce $Kr(\xi,\tau)$, which are new (in thermomechanics):

$$Kr_1(\xi,\tau)=\frac{1}{2}\exp(\tau-\xi)\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}-\sqrt{\tau}\right);$$

$$Kr_2(\xi,\tau)=\frac{1}{2}\exp(\tau+\xi)\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}+\sqrt{\tau}\right);$$

$$Kr_3(\xi,\tau)=\exp(\tau-\xi)\Phi^*\left(\sqrt{\tau-\xi}\right);$$

$$Kr_4(\xi,\tau)=\exp(\tau-\xi),$$

where $\Phi^*(z)=1-\Phi(z)=\left(2/\sqrt{\pi}\right)\int_0^z\exp(-y^2)dy$ is Laplace's function.

Now we find the required solution from (8)–(11) in coordinates (ξ,τ):

$$\sigma_{\xi\xi}(\xi,\tau)=\sigma_{\xi\xi}^{(1)}+\begin{cases}0,(\tau<\xi),\left(t<\dfrac{z-l}{\upsilon_p}\right)\\ \sigma_{\xi\xi}^{(2)}(\xi,\tau),(\tau>\xi),\left(t>\dfrac{z-l}{\upsilon_p}\right)\end{cases}, \qquad (15)$$

where

$$\left.\begin{array}{l}\sigma_{\xi\xi}^{(1)}(\xi,\tau)=-\left[Kr_1(\xi,\tau)+Kr_2(\xi,\tau)\right]\\ \sigma_{\xi\xi}^{(2)}(\xi,\tau)=Kr_4(\xi,\tau)\end{array}\right\}, \qquad (16)$$

for temperature heating,

$$\left.\begin{array}{l}\sigma_{\xi\xi}^{(1)}(\xi,\tau)=-\left[Kr_1(\xi,\tau)-Kr_2(\xi,\tau)\right]\\ \sigma_{\xi\xi}^{(2)}(\xi,\tau)=Kr_4(\xi,\tau)-Kr_3(\xi,\tau)\end{array}\right\}, \qquad (17)$$

for thermal heating, and

$$\left.\begin{array}{l}\sigma_{\xi\xi}^{(1)}(\xi,\tau)=-\left[\dfrac{Bi^*}{Bi^*+1}Kr_1(\xi,\tau)+\dfrac{Bi^*}{Bi^*-1}Kr_2(\xi,\tau)\right]-\dfrac{2Bi^{*2}}{Bi^{*2}-1}Kr_2\left(Bi^*\xi,Bi^{*2}\tau\right)\\ \sigma_{\xi\xi}^{(2)}(\xi,\tau)=\dfrac{Bi^{*2}}{Bi^{*2}-1}\left[\dfrac{Bi^*-1}{Bi^*}Kr_4(\xi,\tau)-\dfrac{1}{Bi^*}Kr_3(\xi,\tau)-Kr_3\left(Bi^*\xi,Bi^{*2}\tau\right)\right]\end{array}\right\}, \qquad (18)$$

for heating by the medium.

Figure 1 presents typical curves for the time dependence of dynamic temperature of stress $\sigma_{\xi\xi}(\xi,\tau)$ in section ξ = 1. Calculations were performed according to (15)–(18). It follows from (15) that only stress constituent $\sigma_{\xi\xi}^{(1)}$ – a longitudinal elastic wave, the front of which moves inward at speed υ_p from the body surface – arises at the beginning in the fixed section. The wave comes at timepoint τ = 1 to section ξ = 1. Stress $\sigma_{\xi\xi}(\xi,\tau)$ increases step-wise passing into the region of the positive (stretching) values at temperature heating

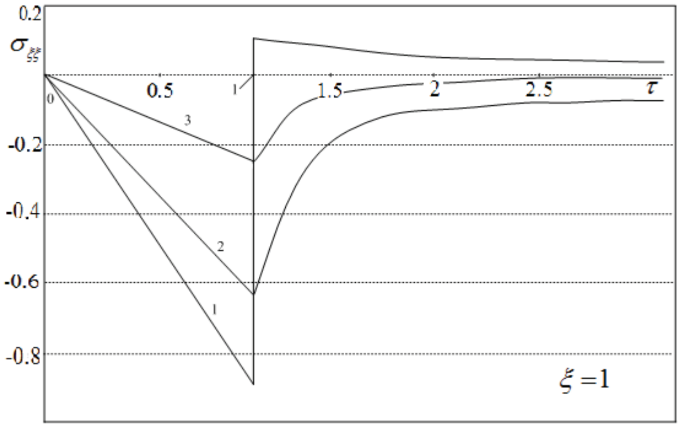

Fig. 1. Change in stress σ_{zz} in time in $\xi = 1$ section:
1 – temperature heating; 2 – thermal heating; 3 – heating by the medium ($Bi^* = 0.5$).

and then quickly decreases to zero reaching quasistatic values $\sigma_{\xi\xi} = 0$. In case of thermal heating and heating by the medium stress changes smoothly, without a jump, continuously, increases while the expansion wave passes, and remains compressing at all $t > 0$. It follows from the curves of Figure 1 that mode (16) (at abrupt temperature heating) is most dangerous as compared to the other modes, (17) and (18). Thus, the propagation of thermoelastic stresses on the basis of the dynamic model is not purely diffusive. Instead, it is due to the propagation of thermoelastic waves.

Relaxation effect and its effect on thermal shock. The stepwise change in the half-space surface temperature from T_0 to T_C ($T_C > T_0$) taken as a basis of the solution is a mathematical idealization that can become almost real only at very large Bio numbers ($Bi = \frac{\alpha}{\lambda_T} l$, α is heat-exchange coefficient). From physical standpoint this is impossible. However, such restriction does not exclude from consideration a large number of thermomechanical problems concerning temperature (abrupt) heating. Nevertheless, in order to study this problem completely let us study the thermoelasticity problem in a case when the surface temperature $T(l,t) = \varphi_1(t)$ increases from T_0 linearly and reaches the value of T_C within a short, but non-zero time range (Fig. 2):

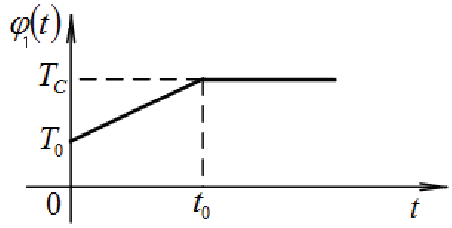

Fig. 2. Change in the temperature of an elastic half-space surface upon heating.

$$\varphi_1(t) = \frac{T_c - T_0}{t_0}\left[t - \eta(t - t_0)(t - t_0)\right] + T_0,$$

where $\eta(z)$ is Heaviside function. In a system of dimensionless coordinates function $\varphi_1(\tau)$ is given by

$$\varphi_1(t) = \frac{\tau}{\tau_0} - \eta(\tau - \tau_0)\left(\frac{\tau}{\tau_0} - 1\right),$$

where $\tau_0 = \upsilon_p^2 t_0 / a$.

Let us find the required functions in the image space (according to Laplace):

$$\left.\begin{aligned} \overline{W}(\xi, p) &= \frac{1}{\tau_0}\frac{1}{p^2}\left(e^{-\xi\sqrt{p}} - e^{-\xi\sqrt{p} - \tau_0 p}\right), \\ \overline{\sigma}_{\xi\xi}(\xi, p) &= -\frac{1}{\tau_0 p(p-1)}\left(e^{-\xi\sqrt{p}} - e^{-\xi\sqrt{p} - \tau_0 p}\right) + \frac{1}{\tau_0 p(p-1)}\left[e^{-\xi p} - e^{-(\tau_0 + \xi)p}\right]. \end{aligned}\right\} \quad (19)$$

Going to the originals we find

$$\left.\begin{aligned} &W(\xi,\tau)=\Psi(\xi,\tau)\equiv\frac{1}{\tau_0}\left[\left(\tau+\frac{\xi^2}{2}\right)\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}\right)-\xi\sqrt{\tau/\pi}\,e^{\frac{-\xi^2}{4\tau}}\right];\left(0\le\tau\le\tau_0\right)\\ &W(\xi,\tau)=\Psi(\xi,\tau)-\Psi(\xi,\tau-\tau_0);\left(\tau\ge\tau_0\right)\end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned} &\sigma_{\xi\xi}(\xi,\tau)=F(\xi,\tau)\equiv\frac{1}{\tau_0}\left\{\eta(\tau-\xi)\left(e^{\tau-\xi}-1\right)+\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}\right)\right\}-\\ &-\frac{1}{2}\left[e^{\tau-\xi}\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}-\sqrt{\tau}\right)+e^{\tau+\xi}\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}+\sqrt{\tau}\right)\right];\ \left(0\le\tau\le\tau_0\right)\\ &\sigma_{\xi\xi}(\xi,\tau)=F(\xi,\tau)-F(\xi,\tau-\tau_0).\ \left(\tau\ge\tau_0\right).\end{aligned}\right\} \tag{21}$$

In the case under consideration stresses change continuously. However, their derivatives with time and with the space coordinate have discontinuities which propagate at speed υ_p .

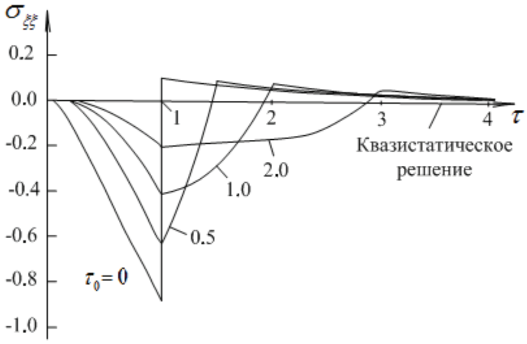

Fig. 3. Change in stress (21) over time in section $\xi = 1$.
[Квазистатическое решение means Quasi-static solution].

Fig. 3 shows plots of time dependence of stress $\sigma_{\xi\xi}(\xi,\tau)$ in point $\xi = 1$ at various values of $\tau_0=\upsilon_p^2 t_0/a$. It can be seen that the stress maximum quickly decreases with increasing τ_0 When $\tau_0=3$, this maximum is only about 14% of its value at $\tau_0=0$ (instant heating). For example, in case of carbon steel ($\nu=0.3;\ G=8\cdot10^9\ n/m^2;\ \rho=7.85\cdot10^3\ kg/m^3;\ a=13\cdot10^{-6}\ m^2/s$) the formula gives expansion wave speed $\upsilon_p=6\cdot10^3\ m/s$, and the dependence between time *t* and dimensionless variable τ is given by $t=3.7\cdot10^{-13}\tau\ s$. When $\tau_0=3$, heating time is $t_0=10^{-13}s$.

For PMMA organic glass ($\lambda=2.26\cdot10^9\ n/m^2$; $\mu=3.8\cdot10^8\ n/m^2$; $\rho=1.2\cdot10^3\ kg/m^3$; $a=1.13\cdot10^{-7}\ m^2/s$) expansion wave speed is $\upsilon_p=1.6\cdot10^3\ m/s$, and t(τ) dependence is $t=0.44\cdot10^{-13}\tau\ s$. When $\tau_0=3$, heating time is $t_0=10^{-13}s$.

These results show that even at so small heating duration the maximum of dynamic stress decreases in comparison with its values at the stepwise change in the body surface temperature.

Cooling. Now let us consider a rather new effect in the theory of thermal shock: cooling of an elastic half-space surface.

So, let us assume that temperature function *T(z,t)* satisfies equation

$$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial z^2};\ \left(z>l;\ t>0\right), \tag{22}$$

initial condition

$$T(z,t)\big|_{t=0}=T_0,\left(\ z\ge l\right) \tag{23}$$

one of the three types of boundary conditions:

$$T(z,t)\big|_{z=l}=T_c;\ \ \left(t>0\right)\ \text{– temperature cooling,} \tag{24}$$

$$\left.\frac{\partial T(z,t)}{\partial z}\right|_{z=l}=\frac{1}{\lambda_T}q_0;\ \left(t>0\right)\ \text{– thermal cooling,} \tag{25}$$

$$\left.\frac{\partial T(z,t)}{\partial z}\right|_{z=l} = h\left[T(z,t)\big|_{z=l} - T_c\right],\ (t>0) \text{ – cooling by the medium,} \tag{26}$$

and boundedness condition

$$\left|T(z,t)\right| < \infty. \quad (z \geq l;\, t \geq 0) \tag{27}$$

where $T_c < T_0$.

Let us introduce dimensionless values:

$$\xi = \frac{\upsilon_p(z-l)}{a};\quad \tau = \frac{\upsilon_p^2 t}{a};\ \mathrm{Bi}^* = \frac{ha}{\upsilon_p};$$

$$W(\xi,\tau) = \begin{cases} \dfrac{T(z,t)-T_c}{T_0-T_c} & \text{(1st and 3rd boundary value problems);} \\ \dfrac{T(z,t)-T_c}{(q_0/\lambda_T)(a/\upsilon_p)} & \text{(2nd boundary value problem)} \end{cases};\qquad \sigma_{\xi\xi}(\xi,\tau) = \begin{cases} \dfrac{\sigma_{zz}(z,t)}{S(T_0-T_c)} & \text{(1st and 3rd boundary value problems);} \\ \dfrac{\sigma_{zz}(z,t)}{S(q_0/\lambda_T)(a/\upsilon_p)} & \text{(2nd boundary value problem);} \end{cases}$$

For function we obtain the following problem:

$$\begin{cases} \dfrac{\partial W(\xi,\tau)}{\partial \tau} = \dfrac{\partial^2 W(\xi,\tau)}{\partial \xi^2}, & (\xi>0;\tau>0); \\ W(\xi,\tau)\big|_{\tau=0} = 1, & (\xi \geq 0); \\ W(\xi,\tau)\big|_{\xi=0} = 0, & (\tau>0); \\ \left.\dfrac{\partial W(\xi,\tau)}{\partial \xi}\right|_{\xi=0} = Bi^* W(\xi,\tau), & (\tau>0); \\ \left|W(\xi,\tau)\right| < \infty, & (\xi \geq 0;\tau \geq 0). \end{cases} \tag{28}$$

(1st and 3rd boundary value problems),

$$\begin{cases} \dfrac{\partial W(\xi,\tau)}{\partial \tau} = \dfrac{\partial^2 W(\xi,\tau)}{\partial \xi^2}, & (\xi>0;\tau>0); \\ W(\xi,\tau)\big|_{\tau=0} = 1, & (\xi \geq 0); \\ W(\xi,\tau)\big|_{\xi=0} = 0, & (\tau>0); \\ \left.\dfrac{\partial W(\xi,\tau)}{\partial \xi}\right|_{\xi=0} = Bi^* W(\xi,\tau), & (\tau>0); \\ \left|W(\xi,\tau)\right| < \infty, & (\xi \geq 0;\tau \geq 0). \end{cases} \tag{29}$$

(2nd boundary value problem).

The solution of problems (28) and (29) is given by

$W(\xi,\tau) = \Phi\left(\dfrac{\xi}{2\sqrt{\tau}}\right)$ upon temperature cooling;

$W(\xi,\tau) = 1 - 2\sqrt{\dfrac{\tau}{\pi}}e^{-\frac{\xi^2}{4\tau}} + \xi\Phi^*\left(\dfrac{\xi}{2\sqrt{\tau}}\right)$ upon thermal cooling;

$W(\xi,\tau) = \Phi\left(\dfrac{\xi}{2\sqrt{\tau}}\right) + e^{2Bi^*\xi + Bi^{*2}\tau}\Phi^*\left(\dfrac{\xi}{2\sqrt{\tau}} + \mathrm{Bi}^*\sqrt{\tau}\right)$ upon cooling by the medium.

The structure of stress is given by

$$\sigma_{\xi\xi}(\xi,\tau) = \sigma_{\xi\xi}^{(1)}(\xi,\tau) + \begin{cases} 0, & \left(\tau<\xi;\ t<\dfrac{z-l}{\upsilon_p}\right); \\ \sigma_{\xi\xi}^{(2)}(\xi,\tau). & \left(\tau>\xi;\ t>\dfrac{z-l}{\upsilon_p}\right). \end{cases} \tag{30}$$

But according to functions introduced above stress components in (30) can be written as:

– in case of temperature cooling

$$\left.\begin{aligned}&\sigma_{\xi\xi}^{(1)}(\xi,\tau)=K_{r_1}(\xi,\tau)+K_{r_2}(\xi,\tau),\\&\sigma_{\xi\xi}^{(2)}(\xi,\tau)=-e^{\tau-\xi};\end{aligned}\right\} \quad (31)$$

– in case of thermal cooling

$$\left.\begin{aligned}&\sigma_{\xi\xi}^{(1)}(\xi,\tau)=K_{r_1}(\xi,\tau)+K_{r_2}(\xi,\tau),\\&\sigma_{\xi\xi}^{(2)}(\xi,\tau)=-e^{\tau-\xi}+K_{r_3}(\xi,\tau);\end{aligned}\right\} \quad (32)$$

– in case of cooling by the medium

$$\left.\begin{aligned}&\sigma_{\xi\xi}^{(1)}(\xi,\tau)=\frac{\mathrm{Bi}^*}{\mathrm{Bi}^*+1}K_{r_1}(\xi,\tau)+\frac{\mathrm{Bi}^*}{\mathrm{Bi}^*+1}K_{r_2}(\xi,\tau)+\frac{2\mathrm{Bi}^{*2}}{\mathrm{Bi}^{*2}-1}K_{r_2}(\mathrm{Bi}^*\xi,\mathrm{Bi}^{*2}\tau);\\&\sigma_{\xi\xi}^{(2)}(\xi,\tau)=-\frac{\mathrm{Bi}^{*2}}{\mathrm{Bi}^{*2}-1}\left[\frac{\mathrm{Bi}^*-1}{\mathrm{Bi}^*}e^{\tau-\xi}-\frac{1}{\mathrm{Bi}^*}K_{r_3}(\xi,\tau)-K_{r_3}(\mathrm{Bi}^{*2}\xi,\mathrm{Bi}^{*2}\tau)\right].\end{aligned}\right\} \quad (33)$$

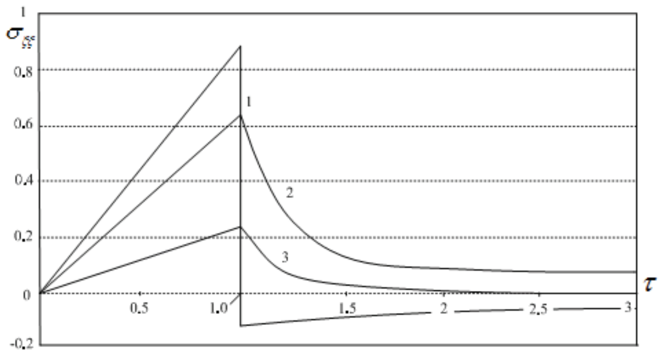

Fig. 4. Stress $\sigma_{\xi\xi}(\xi,\tau)$ dependence on τ in section ξ = 1 in case of temperature cooling (1), thermal cooling (2), cooling by the medium (3) at $\mathrm{Bi}^* = 0{,}5$.

Figure 4 shows dependences of stress $\sigma_{\xi\xi}(\xi,\tau)$ on time τ in section ξ = 1 in various cooling modes calculated according to equations (30)–(31). All the process regularities described above are true in this case as well with the only difference that a compression wave comes to the mentioned section instead of an expansion wave. At the same time these curves demonstrate that the cooling mode creating tensile stress is more dangerous to the medium material than the heating mode, and temperature cooling, as in the case of heating, is more destructive.

Relaxation effect upon cooling. Let us consider a case when the temperature of the surface of an elastic half-space decreases from initial value linearly and reaches final value within a short, but non-zero time range referred to as relaxation time.

$$\varphi(t)=T_0-\frac{T_0-T_C}{t_0}\left[t-\eta(t-t_0)(t-t_0)\right],$$

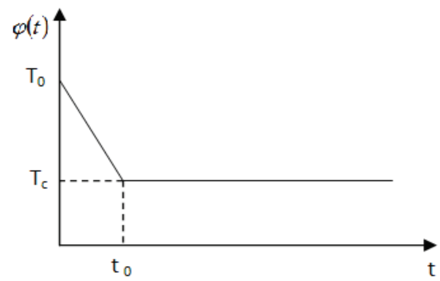

Fig. 5. Change in the temperature of the surface of an elastic half-space upon cooling.

In a system of dimensionless coordinated function is given by

$$\varphi(t)=\frac{\tau}{\tau_0}-\eta(\tau-\tau_0)\left(\frac{\tau}{\tau_0}-1\right),$$

where $\tau_0=\upsilon_p^2 t_0/a$.

Let us find the required functions in the image space (according to Laplace):

$$\left.\begin{aligned}&\overline{W}(\xi,p)=\frac{1}{\tau_0}\frac{1}{p^2}\left(e^{-\xi\sqrt{p}-\tau_0 p}-e^{-\xi\sqrt{p}}\right),\\&\overline{\sigma}_{\xi\xi}(\xi,p)=-\frac{1}{\tau_0 p(p-1)}\left(e^{-\xi\sqrt{p}-\tau_0 p}-e^{-\xi\sqrt{p}}\right)+\frac{1}{\tau_0 p(p-1)}\left[e^{-(\tau_0+\xi)p}-e^{-\xi p}\right].\end{aligned}\right\} \quad (34)$$

Going to the originals we find

$$W(\xi,\tau)=\frac{1}{\tau_0}\left[\xi\sqrt{\tau/\pi}\,e^{\frac{-\xi^2}{4\tau}}-\left(\tau+\frac{\xi^2}{2}\right)\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}\right)\right];\ (0\le\tau\le\tau_0) \quad (35)$$

$$\sigma_{\xi\xi}(\xi,\tau)=\frac{1}{\tau_0}\left\{\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}\right)+\eta(\tau-\xi)\left(1-e^{\tau-\xi}\right)-\right.$$
$$\left.-\frac{1}{2}\left[e^{\tau+\xi}\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}+\sqrt{\tau}\right)+e^{\tau-\xi}\Phi^*\left(\frac{\xi}{2\sqrt{\tau}}-\sqrt{\tau}\right)\right]\right\};\ (0\le\tau\le\tau_0) \quad (36)$$

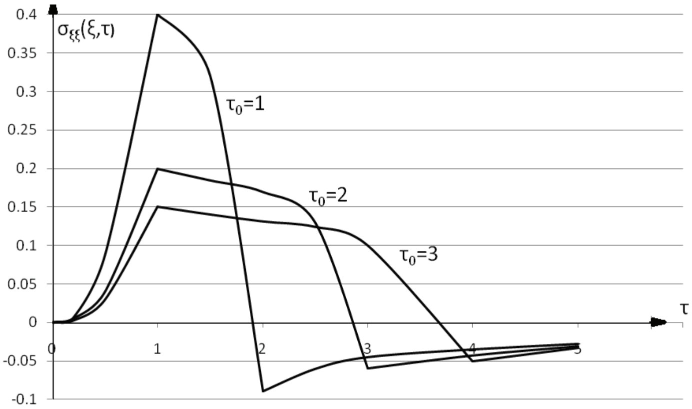

Fig. 6. Time dependence of stress in section ξ = 1 upon different relaxation times.

Here a result about the effect of the rate of a body boundary surface cooling on its thermoelastic reaction is obtained. The longer is cooling time, the lower is tensile stress maximum.

Conclusions

This work considers various thermally stressed states of a massive body arising at various modes of thermal impact on its boundary. It is shown that the propagation of thermoelastic stress on the basis of dynamic model is not purely diffusive. Instead, it is connected with the propagation of thermoelastic waves.

It is shown that abrupt temperature heating is most dangerous in comparison with the other modes. However, if relaxation time is taken into account in calculations (even at small duration of heating), it can be seen that the maximum of dynamic stress decreases in comparison with its values at stepwise temperature change.

It is shown also that the cooling mode creating tensile stress is more dangerous to the material than the heating mode. Temperature cooling, as in case of heating, is more destructive. However, while the cooling time increases, the maximum of tensile stress decreases.

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